Research for Teachers

Secondary School mathematics

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Which aspects of mathematics teaching promote effective student learning and which tend to prevent it?
Students' views on their learning can have much to tell us about effective teaching. For this TLA research summary we selected and summarised a richly detailed, three-year study of the teaching of mathematics, which takes account of the students' own perceptions of their learning experiences.

The study examines in considerable depth approaches to mathematics in two English secondary schools that were using markedly different approaches to the teaching of mathematics. It was conducted in England between 1992 and 1995 and was first published in 1997. The author revised and expanded the original publication primarily to make it accessible to a United States audience.

With the recent developments in the curriculum for 14-16 year olds in Key Stage 4, the study makes a relevant and useful contribution to our understanding of effective mathematics teaching.

The study is:

Boaler, J. *Experiencing School Mathematics*. New Jersey: Lawrence Erlbaum Associates, 2002 (revised edition).

It reports findings on the effect of the different approaches to teaching mathematics on students' academic performance, their beliefs about the nature of 'school mathematics', their self-confidence in mathematics and their ability to use and apply mathematics in the real world. The study shows how specific characteristics of teaching affected boys and girls differently. It found evidence of inequity for students from different socio-economic backgrounds. Most importantly, it uncovers the specific aspects of teaching which had the greatest impact (either positive or negative) on students' understanding.

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Overview

Why is the issue important?
Students' views of the teaching they receive can be very useful for assessing quality of teaching. Determining which aspects of mathematics teaching promote effective student learning and which tend to prevent it is an important tool for learning about effective teaching.

What did the research show?
Students at the 'progressive' school:

- understood mathematics better
- were able to apply their knowledge better
- were more confident in new mathematics-related situations
- remembered their mathematics better
- did better at GCSE mathematics.

than students at the 'traditional' school.

How was this achieved?
Mathematics teachers at the 'progressive' school put students into mixed ability groups, gave open-ended tasks, encouraged students to discover and use their own mathematical methods, gave students a high degree of choice with regard to the direction of their work, asked open questions that required students to think hard about their answers as well as how they had arrived at them, and encouraged discussion between students. They aimed to give students a comprehensive mathematical understanding that would help them throughout life, rather than narrowly focussing on examination success. Students at the 'traditional' school were placed in ability groups at the start of Year 9. Lessons typically followed a consistent pattern, were highly structured, used set approaches to problems, and required students to give only brief responses to questions.

How was the research designed to be trustworthy?
The study tracked a cohort of around 300 students in two English secondary schools as they progressed through Years 9-11. The two schools were similar in terms of the student intake and socio-economic conditions, but the schools differed markedly in their philosophy and ethos, approach to classroom organisation, and teaching methods in mathematics in Key Stage 4.The researcher collected a variety of student test data during each year of the study, surveyed and interviewed the students, interviewed teachers and observed around 100 lessons.

What are the implications?
The study showed the importance of:

- giving students opportunities to work through problems that require thought and to reward students who demonstrate careful mathematical thinking
- opportunities to discuss with each other what they can do if they get stuck
- using open-ended activities and prompting students to discuss the problem fully and to encourage them to record it in their own mathematical language
- helping teachers to be aware of their preferred teaching techniques and the effects these have on their students.

What do the case studies illustrate?
The case studies show, for example:

- a mathematics classroom was set up to enable the students to learn algebra more effectively via an open-ended, investigative style of teaching and learning
- whole class teaching can be used effectively to improve the quality of students' learning in mathematics

- some of the difficulties students experience and the misconceptions they may adopt in algebra if the subject is taught in a way that emphasises routine skills rather than understanding, and if explicit attention is not paid to the careful use of technical language.

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Study

What did the study find out about learning in mathematics in the 'traditional' and 'progressive' schools?

The study examined the markedly different approaches to mathematics teaching at two schools and the effect on students in terms of their:

- performance on different types of assessment
- beliefs about mathematics
- attitudes to mathematics
- enjoyment of the subject.

It is important to start by saying that differences in the ethos of the two schools extended beyond the mathematics department. The schools differed in their dress codes for students and staff, student behaviour within and outside lessons, student-staff interactions and the persona of the head teacher. The researcher described Amber Hill as having an ethos that induced obedience and conformity. Students were well behaved and quiet in both lessons and corridors. They received annual grades on their cooperation and "the wearing of school uniform". Within lessons, communication between the staff and students was teacher led but friendly. Students described teachers as approachable and helpful. The head teacher, who was described by the researcher as 'austere' in appearance and 'authoritarian' in his approach to decision making, seemed to want the school to be seen as academic and respectable.

Phoenix Park had a long tradition of progressive education, a commitment to equality of opportunity and a thriving special educational needs department. Many subject departments used a project-based, problem-solving approach with little recourse to textbooks. All subjects were taught in mixed ability groups. The researcher described Phoenix Park as 'relaxed' in general ethos and standards of dress - the students did not wear uniform and the teachers mostly dressed casually. It had a calm atmosphere, as students walked slowly round the school and there was no running or shouting. Staff and student conversation was frequent and informal and there was an emphasis on student self-reliance and independence. The head knew all of the students by name and they seemed comfortable in his presence.

The two schools showed no significant differences regarding sex, ethnicity or social class and the student cohorts at the beginning of Year 9 reached very similar levels of attainment on a broad range of mathematics tasks. However, the subsequent learning experiences of Amber Hill and Phoenix Park students were very different. Amber Hill students encountered mathematics within a 'traditional', class taught, transmission model of teaching. The students at the 'progressive' school, Phoenix Park, worked on open-ended activities in an informal environment and learned about new mathematical methods and procedures only when they needed to use them in projects. Readers might like to read a case study that gives an example of such 'just in time learning' (case study 1) set within the context of an open-ended approach to teaching algebra.

The study went beyond simplistic analysis of 'traditional' and 'progressive' methods of teaching and classroom organisation. It set out to discover whether and how students' learning was affected because they encountered new ideas in different forms and contexts. It examined what specific aspects of teacher practice and belief led to particular outcomes.

Over the course of three years, the researcher, Jo Boaler gathered a wealth of detail on teaching methods, classroom organisation and relationships between teachers and students. She used many, varied assessments to find out whether differences existed in the extent or nature of students' understanding as a result of their experiences. These are explained more fully in the page on study design. The assessments showed that the students at the 'progressive' school:

- understood mathematics better
- were able to apply their knowledge better
- were more confident in new mathematics-related situations
- remembered their mathematics better
- and did better at GCSE mathematics than students at the 'traditional' school .

What GCSE results did students taught by 'traditional' and 'progressive' methods achieve?
Phoenix Park students outperformed Amber Hill students at GCSE despite the fact that teachers at Amber Hill made it their top priority to prepare their students for this examination. Phoenix Park teachers, on other hand, stated that their approach to mathematics was not narrowly targeted at examination success. Although the GCSE examinations were important to the students and the school, teachers allowed students to select and enter unfinished coursework for the examination, for example.

At the end of Year 11, Amber Hill entered 84 per cent of their students in the GCSE examination and Phoenix Park entered 94 per cent of theirs. The results were as follows:

- more Phoenix Park students passed the examination (gained A*-G grades) than Amber Hill students
- more of the Phoenix Park students (2.9 per cent) than Amber Hill students (0.5 per cent) gained the highest Aor A grades
- similar proportions of the students at the two schools gained A*-C grades
- Ato C results for both schools were lower than the national average, but this might be expected from the intakes of the two schools
- the proportion of Ato G passes at Phoenix Park was higher than the national average for the examination, suggesting the students made better progress than the national average.

The performance of pupils at Phoenix Park was closer to early measures of their ability in mathematics than was the case at Amber Hill. The researcher compared the results of NFER-Nelson tests of mathematical ability which students all took at the end of Year 8 with their eventual GCSE grade. She plotted the one measure against the other on scatter-grams and found that there was a much closer match between the initial measure of a student's ability and their eventual result at GCSE at Phoenix Park than at Amber Hill. A perfect correlation (an exact match) would be equal to one. The correlation for the Phoenix Park scatter-gram was 0.67, whilst the correlation for Amber Hill was only 0.48.

This meant that fewer students at Amber Hill achieved the results expected of them than was the case at Phoenix Park, where students' GCSE results were more closely linked to early measures of ability. At Phoenix Park, there were a few students whose GCSE results did not correlate closely with their prior performance - several of these performed better at GCSE than expected. At Amber Hill, a greater number of students gained disappointing GCSE results that did not match their previous levels of performance.

The Amber Hill students who did badly, despite being high achievers at the end of Year 8, tended to be working class. This was not true of Phoenix Park, where the very small number of underachieving students came from a wider range of social classes.

What characterised mathematics lessons in the 'traditional' school?
The two schools studied were similar in terms of the student intake and socio-economic conditions. Their students had similar previous experiences of mathematics, working through individualised workbooks at their own pace, with little whole class teaching in Years 7 and 8. However, the schools differed markedly from one another in their:

- philosophy and ethos
- approach to classroom organisation
- teaching methods in mathematics in Key Stage 4

- views on the relative importance of examinations.

The students behaved differently in mathematics classes within the two schools at Key Stage 4. This could be seen in:

- their responses to questions
- the time they spent on task
- their level of engagement with the work.

The students at the 'traditional' school (Amber Hill) were placed in eight 'sets' or ability groups at the start of Year 9. The teachers in the Amber Hill mathematics department used a text book approach to their lessons. They were all mathematics specialists. Mathematics lessons at Amber Hill:

- followed a consistent pattern
- were highly structured
- used set approaches to problems
- were typified by a concern to cover the national curriculum and prepare students for the GCSE examinations
- required students to give only brief responses to questionsshowed good student behaviour and motivation.

Teaching approaches at Amber Hill

Consistent lesson structure
Teachers at Amber Hill were all well qualified mathematics specialists. They were friendly and developed good relationships with students. The students reported that they found the teachers approachable and helpful. All the teachers adopted the same approach to teaching mathematics. They explained methods from the front of the class for the first fifteen to twenty minutes, often questioning pupils using short, closed questions. Teachers then gave the students questions from textbooks to work through. During this time, they moved between pupils to help them and to check progress.

From Year 9 the students used the same textbooks in almost every lesson. They completed assessments of their work at the end of each chapter. Lessons lasted an hour and the format did not vary, except for three weeks of Years 10 and 11 when the students completed an open-ended investigation.
Set approaches to problems

Amber Hill mathematics teachers generally offered students only one method of approaching a problem. They presented these in a clear, structured way, but did not discuss their choice of method or when and why they worked. They did not encourage students to discuss alternative approaches or to try their own methods. As the following comment shows, teachers chose this approach because they thought it was an efficient way of getting students through the curriculum.

"You're very stringent to a time limit, you haven't got the time...there's certain things you have to sit down and tell them. I could spend a week letting them work through on their own, or, I know this group, I could explain it to them in one lesson and they'd understand it. Which one do you do?" (Edward, Amber Hill, teacher)

Teachers at Amber Hill were conscientious and their conversations often indicated a concern to complete the syllabus.

The restriction of student responses
The vast majority of the students' experiences of mathematics involved short, procedural activities. Amber Hill mathematics teachers used closed questions requiring only brief responses. They often simplified mathematical principles or methods to a set of rules or procedures for students to learn. Although the textbook offered open questions towards the end of each exercise, the students were rarely allowed to work through these alone. Instead, teachers would break such questions into smaller parts and guide students through the mathematical decision making, either on a one-to-one basis or with the whole class. Some teachers put demanding questions up on the board before lessons and went through them during the introduction. After extensive classroom observations, the researcher judged that students were given little time to work on

activities that made them think.

The students reported in interviews that they were actively discouraged from using their own methods.

"In school you have to use the method you are told to."
(Danielle, Amber Hill, Year 10, Set 2)
"Normally there's a set way of doing it and you have to do it that way. You can't work out your own way so that you can remember it."
(Carly, Amber Hill, Year 11, Set 1)

Student behaviour
Amber Hill students sat in pairs, but tended to work alone, except for checking answers with their partners. They were very well behaved and confined any misbehaviour to chatting with their partners. They appeared to spend almost all their time on task. In ten representative lessons, the researcher found that 100, 99 and 92 per cent of the students were on task ten minutes into the lesson, half-way through and ten minutes before the end of the lesson respectively. The Amber Hill students believed mathematics to be a very important subject and wanted to do well in it.

What characterised mathematics lessons in the 'progressive' school?
The mathematics teachers in the 'progressive' school, (Phoenix Park) taught mixed ability classes using open-ended activities which allowed the students to respond at different levels and thus were differentiated by outcomes. Mathematics lessons at Phoenix Park:

- were varied and used open-ended tasks until the start of GCSE preparation in Year 11
- gave students a high degree of choice with regard to the direction of their work and encouraged them to discover and use their own mathematical methods
- aimed to give students a comprehensive mathematical understanding that would help them throughout life, rather than narrowly focussing on examination success
- asked open questions that constantly required students to think
- were very informal, encouraged discussion, allowed a degree of off-task activity and elicited varying levels of student motivation.

Teaching approaches at Phoenix Park
The teachers' concern with getting through the curriculum that was typical of Amber Hill was absent in Phoenix Park. The school's scheme of work was described as "very sparse", but the activities were carefully chosen to provide different access points for different students. Mathematics teachers at Phoenix Park were well qualified and all except one teacher (whose degree was in engineering) were mathematics specialists.

Fluid lesson structure
When they joined Phoenix Park in Year 9, the students were placed in mixed ability groups. From then until half way through Year 11, the students worked on open-ended projects. (In January of Year 11, the students started preparing for their GCSE examinations and the lessons at Phoenix Park became more structured and directed.) During the two and a half years of open-ended work, the teachers introduced students to a project or theme and then allowed them to explore this using their own ideas and mathematical knowledge. Projects usually lasted about three weeks. Students exercised a high degree of choice in how they approached their work.

"We're usually set a task first and we're taught the skills needed to do the task and then we get on with the task and we ask the teacher for help."
(Simon, Phoenix Park, Year 11)

Open approach to problems
Teachers introduced activities which they knew were mathematically rich, but they did not have fixed ideas about how students would interact with the problems. They did not give students specified paths through their activities and, when asked questions, seemed to make deliberate efforts not to tell the students what to do or to give them easy answers. For example, a teacher refrained from giving information about shapes to a student,

referring him instead to a mathematics dictionary where he could look up the information.

The deepening of student responses

Teachers encouraged pupils to think about their work and to trust their common sense. Students were encouraged to estimate and to compare results with their expectations. One student commented:

"First of all you have to try and find your own methods, then if you really get stuck the teacher will come and give you suggestions for stuff and tell you how to, like, progress further and then you can kind of think about it."
(Andy, Phoenix Park, Year 11)

Teachers consistently asked students to expand on their answers, to explain their reasoning and to communicate their thought processes within their groups and sometimes to the whole class. Finding an answer was not sufficient; students were encouraged to think about why a particular approach worked and to explore alternative approaches.

Student behaviour

Mathematics lessons at Phoenix Park were often described by students as "noisy". The researcher commented that in one hundred or so lessons she observed at the school, typically one third of the students would be wandering around the room, chatting about non-work issues. Some students remained off task for long periods of time. Others drifted on and off task. In eleven representative lessons, the researcher found that 69, 64 and 68 per cent of the students were on task ten minutes into the lesson, half-way through and ten minutes before the end of the lesson respectively. It was difficult for staff to monitor the amount of work completed by students during the course of each project. The teachers trusted the students to get on with their work, spent their time helping students who wanted help and left others to their own devices, unless they became disruptive to others. Practitioners who aim to get the best from all students, including those with a low level of personal motivation, may find this both surprising and unacceptable. Nevertheless, the study found evidence that students at Amber Hill perceived themselves to be off task for similar periods of time to the Phoenix Park students - they merely looked as if they were working. When the Phoenix Park students chose to work, they did so with interest.

How did the two groups of students compare in tests of applied mathematics?

Phoenix Park students did better on tests of their applied knowledge than students at Amber Hill. This might be expected, as the Phoenix Park method of teaching required them to apply their knowledge to realistic contexts to a much greater extent than was the case at Amber Hill. In the findings reported below, the Phoenix Park students were drawn from the full ability range, as all classes were of mixed ability. The Amber Hill students were drawn from the top half of the school's ability range. (The researcher decided early on to focus some, but not all, data collection on sets 1 to 4, partly because the department head was most comfortable with her visiting those classes and partly because those sets showed interesting performance patterns in an early applied activity.)
Findings from the first year of the study

At the end of Year 9, the study tested about half the students in each school on multiplication, division, area, volume, percentages, angles and measurement. All students had met these mathematical areas in class. A few weeks later, the same students tackled an architectural activity that required them to apply the same mathematical knowledge. To succeed on the activity, students needed to choose, combine and use different mathematical methods. They could use calculators at any stage. The study found that Phoenix Park students were slightly less successful than Amber Hill students on the test questions, but much more successful on the activities.

In the activities using volume:

- more Phoenix Park students (75 per cent) solved the problem correctly than Amber Hill students (55 per cent)
- of those students who answered relevant test questions incorrectly, more Phoenix Park students (29 per cent) than Amber Hill students (11 per cent) nevertheless did well on the practical activity
- more Amber Hill students (28 per cent) could not use relevant mathematics in the activity, despite answering test

questions correctly, compared with Phoenix Park students (16 per cent).

In work on angles:

- of those students who correctly estimated an angle in the test, more Phoenix Park students (83 per cent) than Amber Hill students (62 per cent) were also correct in the applied activity
- the least successful Amber Hill students came from the top set, many of whom tried unsuccessfully (and inappropriately) to use trigonometry to estimate an angle.

Findings from the second year of the study
One year later, all of the students in the top four sets at Amber Hill and all students from four mixed ability classes at Phoenix Park were given a second activity and related tests. The tests took place a month before the activity and assessed all the mathematics featured in the activity. In the first hour of the applied test, students had to plan and draw to scale rooms, doors and furniture on the empty plan of a flat, following various 'building regulations'. During a second, hour long lesson, they had to answer two questions in relation to their flats.

Many more Phoenix Park students (61 per cent) than Amber Hill students (31 per cent) gained the highest grade, despite being taken from a wider attainment range. To receive this grade, they had to produce well-planned designs, with appropriately sized and scaled furniture.
Nearly one quarter of Amber Hill students drew rooms of an inappropriate size or included wrongly scaled furniture on the activity despite the fact that only one tenth of them failed to use scale correctly in the short, written test.

The scores are given below.

Flat design results						
	Percentage of students gaining each grade					Total number
	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	
Amber Hill	31	24	7	18	19	99
Phoenix Park	61	6	8	13	12	89

When asked to estimate the approximate cost of a carpet to cover the whole of their flat, over one third of Amber Hill students attempted to find an exact measurement of floor space. Later, in interviews, some of these students said that they used this degree of accuracy because they thought they had to "show their working". The researcher commented that they were probably doing what they thought was expected of them, which meant working with the numbers and ignoring the wider context. The Amber Hill students' performance showed that they had difficulty making use of the mathematics they had learned in an applied situation, not due to a lack of mathematics knowledge, but due to the ways in which the students interpreted the demands of the activity.

How did the two groups of students perform in tests of long-term learning?
The students at Phoenix Park outperformed students at Amber Hill in assessments designed to measure long-term learning. In this assessment, students were tested on a piece of their school work just before being taught the work, immediately after completing it and then six months later. On each of these occasions, students took exactly the same test.

The results showed that, immediately after their lessons both sets of students could remember a reasonable proportion of their work. Six months later the Phoenix Park students remembered far more of what they had learned than Amber Hill students.

Phoenix Park Year 10 group had forgotten one sixth of what they had learned
Phoenix Park Year 9 group had forgotten one third of what they had learned
Amber Hill Set 4 Year 10 had forgotten half of what they had learned
Amber Hill top set Year 9 had forgotten two thirds of what they had learned.

The tests the Amber Hill students took were exact replicas of their exercise book questions with different numbers of contexts. The test designed for Phoenix Park could not be an exact replica of their class work, as the students were working at different levels of mathematics. Despite this, the Amber Hill students did less well on the tests of long term learning than Phoenix Park students.

The first group at Phoenix Park was a Year 9 group working on long division without a calculator. It was approached in an unusually didactic way for Phoenix Park: one method was introduced on the board and then the students explored division patterns. The work lasted for two lessons. The Year 10 work, taught by the same teacher, was a more typical Phoenix Park project on statistics lasting about three weeks.

The two groups assessed at Amber Hill were a Year 9 top set and a Year 10 Set 4, both taught by the same teacher. In both groups, the teacher explained the methods at the board and then the students practised exercises. The top set was taught at a fast pace, as was normal for the school. Both groups worked for about three weeks on the topics assessed. The group of most able students forgot more in the tests than any other group.

How did preparation for GCSE examinations differ in the two schools?

From January of Year 11, the teaching and learning environment at Phoenix Park changed to become more like that of Amber Hill, as teachers prepared students for the GCSEs. Phoenix Park students were grouped into three bands according to the examination they were entered for - foundation, intermediate, or higher. The teachers used blackboards more frequently to explain procedures and the students practised procedures within textbook questions, worksheets and past examination questions. Students did not appear to be worried about the approaching examination. Many reported that they did not bother to revise for the examination.

"I can't say anyone I know is bothered about their GCSEs. I don't think we're revising or bottling down or anything. I think it hasn't hit us yet."
(Helen, Phoenix Park, Year 11)

At Phoenix Park, the teachers explicitly stated that their intent in mathematics was to give students a comprehensive understanding that would help them in life. They were not aiming solely at success in GCSE. Nevertheless, success in the examination was important to them and to the school. It is therefore surprising that teachers at Phoenix Park did not provide some students with calculators on the day of the examination, given that many students could not afford to buy their own. The mathematics department claimed that it did not have the money to replace calculators lost during the year.

"Like the day before they told us all the equipment we needed and we had to go out and buy it and if you didn't have any money then you didn't have the equipment."
(Linda, Phoenix Park, Year 11)

The researcher was given special permission to analyse the completed GCSE papers at both schools. She found that six Phoenix Park students wrote, "I haven't got a calculator," on their papers and, at frequent points in the examination, wrote out the method they had used in the questions, thereby making the best of a difficult situation, but did not calculate the answers. This lost them marks.

Whereas Phoenix Park teachers rarely mentioned the examinations, at Amber Hill, the teachers constantly stressed its importance and reminded students of the need to prepare for it. They were clear that preparing students to do as well as possible at GCSE was their most important aim. The students' ability to use mathematics in more realistic situations was not as important to them. Amber Hill students shared their teachers' desire for GCSE success and even those who disliked mathematics worked hard to achieve a good result. When asked why he worked so hard in lessons despite having stated a dislike for mathematics, one student replied:

"Because we want to do well. Mathematics GCSE is really important - everybody knows that."
(Chris, Amber Hill, Year 11, Set 4)

The researcher commented that the pressure to do well at GCSE may have disadvantaged the Amber Hill students, just as lack of pressure may have disadvantaged some Phoenix Park students. Phoenix Park students also faced the real difficulty of taking the examination with no calculator. Yet their GCSE results were better, on both raw and value added results, than those of the more 'traditional' school. Both schools started with pupils at similar levels of achievement and attainment, as measured on a broad range of criteria, and Phoenix Park students achieved more Aand A grades and more Ato G grade passes.

What effects did different teaching styles have on student beliefs and attitudes?
The study explored the relationship between different classroom interactions and the understandings, beliefs and dispositions that students subsequently developed.

It found that students offered a programme of study in which they had to think hard about open-ended problems were more likely to:

- believe that mathematics was of relevance to everyday life
- believe that understanding in mathematics was more important than working fast or getting to a correct answer
- enjoy mathematics
- show confidence and creativity in mathematics, especially when encountering new situations in which to apply their knowledge

than students who had experienced highly structured lessons that taught rules and procedures to the whole group at a fixed pace.

Moreover, at the more 'traditional' school, the study uncovered marked differences between girls and boys in their response to mathematics.

In the next few pages we explore reports of the relationship between mathematics experiences at the two schools and student beliefs and attitudes.

What were student beliefs about 'school mathematics' and mathematics in everyday life?
Students at both schools were asked to rank different aspects of mathematics in terms of their importance. These included:

- working at a fast pace
- getting a lot of work done
- remembering rules and methods
- knowing how to use a calculator
- understanding mathematics.

The student responses showed that the girls at Amber Hill were closer in their beliefs about mathematics to Phoenix Park students than to the boys at Amber Hill. The male Amber Hill students were more likely to believe that working fast and remembering rules were important, whereas girls at Amber Hill and the Phoenix Park students were more likely to believe that understanding was important.

Students at 'traditional' Amber Hill were taught standardised methods for reaching solutions to a problem. They tended to express beliefs that:

- students needed to learn rules and apply them
- it was important to answer questions in "the way the teacher wants you to"
- memory was more important in mathematics than thought.

These beliefs had several consequences for Amber Hill students:

- they were more likely to attempt to remember and apply rules than to try to interpret and understand what they were doing
- when they encountered questions that did not require simplistic use of a rule, they asked for help, gave up, or tried to use a rule inappropriately
- they could apply learned rules to familiar situations but became easily confused when faced with a new situation.

The study found evidence that the two different sets of students varied in their ability to connect school mathematics to real life situations. Both sets of students were asked to describe situations in which they had used mathematics outside school. The Phoenix Park students mentioned various activities such as organising a bank account, route-planning for a paper round, reading the scale of a map, laying slabs in the front garden and sorting out a magazine collection, as well as situations using data handling, money or shopping. The responses of Amber Hill students were far less varied and three-quarters of them confined their responses to using money or shopping.

Amber Hill students found it difficult to see the relevance of 'school mathematics' to mathematical situations in the real world. One commented:

"Well, when I'm out of school, the maths from here is nothing to do with it...most of the things we've learned in school we would never use anywhere."
(Richard, Amber Hill, Year 11, Set 2)

Students at Phoenix Park saw no distinction between the mathematics they learned in school and the mathematics of the real world. They said they could use the same methods in both places.

"I'd probably try and use what I've learned in school."
(Gary, Phoenix Park, Year 11)

Readers may wish to read a case study of how data generated from real problems was used by students to develop their mathematical skills in cross-curricular projects(case study 2).

Once Phoenix Park students began formal GCSE preparation, their perception of a link between 'school' and 'real life' mathematics declined.

"I wouldn't be able to use the stuff now [examination preparation] because I don't understand it."
(Linda, Phoenix Park, Year 11)

How much did students at 'progressive' and 'traditional' schools enjoy mathematics and why?

The Phoenix Park students were more positive about mathematics lessons than the students at Amber Hill. At Phoenix Park, 43 per cent of the students offered positive or very positive comments on mathematics lessons, compared with only 23 per cent of Amber Hill students. The situation was reversed for negative comments, with only 25 per cent of Phoenix Park students offering negative or very negative comments compared with 39 per cent of Amber Hill students.

The Phoenix Park students varied in their enjoyment of mathematics. About one third of them were consistently positive. They felt that the work was varied and appreciated the freedom of choice they had to tackle it. About half of the year group said they enjoyed mathematics most of the time. Their degree of enjoyment depended upon the specific project. About five students in each class at Phoenix Park disliked and resisted the open nature of the work. They complained that they were left on their own without knowing what to do. These students were mainly boys and they were often disruptive, not only in mathematics, but across the school. The behaviour of these students tended to improve as they got older. By the time they were higher up the school, they had become more accustomed to having to think for themselves and complained vehemently about the closed nature of the work taught by a student teacher.

Students from both schools agreed that they enjoyed variety, doing practical, open-ended work and working at their own pace. However, the students at Amber Hill complained that their mathematics rarely offered these features. They disliked always working from the same text book and having to practise many of the same types of problem. The sole break from the unvarying pattern of Amber Hill mathematics lessons was three weeks of open-ended coursework completed in the year prior to GCSEs. These open-ended tasks involved mathematical decision making in "real-life" situations, like planning a trip, and offered all three of the features students said they enjoyed. In addition, the tasks allowed students to discuss their work in cooperative groups.

When asked to describe "the most interesting piece of mathematics" they had ever encountered, Phoenix Park students listed a wide variety of projects. The Amber Hill responses lacked variety. The majority of Year 9 students, who had not yet experienced coursework, cited the same lesson: using Logo, a programming language on the computer. Most of the older Amber Hill students chose the coursework as their most interesting mathematics experience. They said they found it difficult and had to think hard, but they felt proud of what they had achieved by the end:

"It's a better way to learn 'cause I can figure it out for myself. The books just, it's too much leading you through it."
(Sacha, Amber Hill, Year 11, Set 4)

"You feel more proud of the projects when you've done them yourself. If it's just working through the book, you can't feel proud ... but if it's a big project ... and you've worked hard and if you get a good mark you feel really good about it."
(Lola, Amber Hill, Y11, Set 3)

What gender differences did the study find?
Surprisingly, the study found no significant differences between male and female attitudes to, or enjoyment of, mathematics at Phoenix Park. This finding is unusual, as such gender differences in attitudes to mathematics have often been found by other studies. However, gender differences in approaches to mathematics emerged in the more 'traditional' mathematics department. Amber Hill boys and girls differed with regard to:

- the reasons they gave for enjoying working at their own pace
- the reasons they gave for enjoying course work
- their attitudes towards working fast
- their expressed needs to understand the subject
- their attitudes to working and discussing problems in groups
- their ability to experience rewards and satisfaction in their mathematics lessons
- their level of unhappiness and stress.

Both girls and boys at Amber Hill strongly preferred the individualised work in Years 7 and 8 and the course work in Year 10, but for different reasons. The girls wanted to understand what they were doing before they moved on to something else, whereas the boys wanted to work fast. The following comments on individual working, one from a girl, one from a boy, were typical:

"We had time to read it through and if we didn't get it, we had time to read it again, but like with this, we can only read it through once because she wants us to hurry up and finish it."
(Lindsey, Amber Hill, Year 11, Set 4)
"It was better then, weren't it? We used to compete... We could do it at our own pace and we used to be books ahead of the others."
(Christopher, Amber Hill, Year 11, Set 4)

The Amber Hill girls said they enjoyed their coursework because, apart from having more time to think about and tackle the work, they could discuss it in cooperative groups and these discussions helped them to understand. Many boys reported enjoying their course work but said this was because it made a change from their usual work. Few said that it gave them opportunities to think, use their initiative, or understand better. Boys rarely mentioned group work. One who did so disliked it because he thought it slowed him down.

Boys and girls at Amber Hill gave statistically significant different responses to the features of mathematics they thought were important:

- girls were far more likely than boys (91 per cent versus 65 per cent) to see understanding as the most important aspect of learning mathematics
- boys were far more likely than girls to see remembering rules and methods (24 per cent versus 4 per cent) or getting a lot of work done (19 per cent versus 5 per cent) as very important in mathematics.

These different perceptions of the relative importance of understanding and "getting through the work" affected boys' and girls' chances of feeling satisfied with their mathematics. Boys and girls used different strategies when they found their work difficult. When confronted with difficult aspects of mathematics, boys were more able than girls to set aside their desire for understanding, choose a mathematical procedure they thought might work, crank mechanically through some calculations and move swiftly on to other questions where they hoped for greater success. Girls tended to struggle on with the same question, growing more frustrated and unhappy. The Amber Hill girls reported high levels of anxiety about mathematics.

The emphasis at Amber Hill upon covering the curriculum, on working fast and on applying rules in a mechanical way all served to set girls who needed to understand their work at a disadvantage. In some cases, this approach affected not only the girls' happiness but also their eventual level of attainment.

How did the different approaches to mathematics affect students' self-confidence, creativity and levels of engagement with the work?

The students at the 'traditional' school were well behaved and most seemed to be on task most of the time. However, when they were asked to write down anonymously how many minutes they worked in the one-hour long lessons, the average time they gave was only 38 minutes. Student perceptions of time on task were far lower than their apparent behaviour suggested. The students looked as if they were working but reported that they were not. Many said that they "switched off" in mathematics lessons or worked without really thinking about it. They knew a good mathematics qualification was important and resigned themselves to working, but did so mechanically.

"People think, oh well, I've got to do it so I might as well do it."
(Carly, Amber Hill, Year 11, Set 1)
"Just like a computer, you'll do it, but when you get the answer, you won't be sure that it's right...you'll be like - this is how we learned it, but is it the answer? You're never certain."
(Simon, Amber Hill, Year 11, Set 4)

Students at the 'progressive' school were encouraged to be independent and responsible for their own behaviour. They chatted or "messed about" for some of the time but, when they chose to work, did so in a focussed way. The students were given a great deal of freedom to find their own solutions in mathematics and their responses were discussed openly with their peers and in a non-judgemental way by staff. This seemed to have positive effects on their self-confidence and ability to offer creative solutions to problems.

The study assessed students' ability to apply mathematics in an activity called "Planning a Flat". The Phoenix Park students showed greater originality and imagination in their responses than the Amber Hill students. In the 89 designs produced by Phoenix Park students, there were 35 unusual rooms, including a variety of games, sports and play rooms, studies and computer rooms, utility and store rooms, a Jacuzzi, cocktail bar and piano room. By comparison, the 99 designs produced by Amber Hill pupils included only six unusual rooms: two pool rooms, two swimming pools, a play room and a store room.

The study showed that 82 per cent of Phoenix Park students but only 65 per cent of Amber Hill students agreed with the statement "It is important in mathematics to use your imagination". Perhaps the Amber Hill students felt less confident that they had permission to be creative.

How did the 'progressive' mathematics department differentiate work for groups of mixed ability?

At Phoenix Park, students remained in mixed ability groups until shortly before taking GCSEs when they were placed in three groups for examination preparation. The mathematics teachers there achieved something that many people think is not possible. They taught a wide range of students together in the same classes and provided stimulating and appropriate experiences for each or most of them. They did this by paying careful attention to the different learning experiences that students needed.

The main strategy they used was differentiation by outcome. Phoenix Park staff planned work that offered students a choice of different starting points, activities and routes through. These different activities led students to encounter a variety of concepts and to develop different skills and understandings.

The second strategy that they used was differentiation by task (making different work available to different students). The way in which they did this was quite subtle. Although students chose what to do and how to do it, throughout each project, the teachers discussed the students' work with them, suggesting possible additional lines of enquiry and steering able students to more demanding and mathematically rich aspects of the project.

Phoenix Park mathematics teachers believed that the open-ended approach they used was valuable for all students and that it was their job to make the work equally accessible. They did so in various ways.

Support at the start of projects
Phoenix Park teachers always introduced each new problem to the whole class of students and discussed it with them in some depth. They never left students to interpret text-based problems alone. They had further discussions with groups and with individuals to clarify their understanding and to check each student had some idea of how to start their investigation. The following quote from a teacher took place at the end of a long, introductory discussion of a new project in which the pupils had played an active part:

"So, I'm interested in area. I'd like you to explore these shapes and find areas. Now, the first thing I'd like you to do is to record what I've been talking about. My writing isn't sufficient; you need to put things in your own words, your own version of the problem. Expand it, write what it means, pick out shapes, decide what order you need to do them in!"
JC, Year 9 class, Phoenix Park

They provided resources, such as mathematics dictionaries and practical apparatus that the students could use independently to explore their emerging ideas or to increase their knowledge on a 'need to know' basis. Staff consciously avoided giving the students easy answers or providing them with a lot of structure when they were struggling.

Checking students' understanding and uncovering misconceptions
The teachers provided scaffolding through appropriate help, questioning and instruction. The Phoenix Park mathematics teachers often encouraged students to explain their methods and reasoning and justify the approach they used. This helped them to detect and explore student misconceptions and help them to gain a deeper understanding. It also offered students an opportunity to think out loud and clarify their understanding as they struggled to put their findings into words. Phoenix Park teachers had a good awareness of what their students did and did not understood and they used this information to support them as they tackled the projects. They:

- asked insightful questions of each student
- listened to the response
- determined what the response said about the student's thinking
- based further questions and guidance upon what they had discovered about the student's thinking.

Clarifying expectations
The Phoenix Park teachers made it clear that finding an answer was insufficient. Good mathematics was about communicating the thinking behind the result.

"Brilliant work, John, but you can't just write it down, there must be some sense to why you've done it, some logic. Why did you do it that way? Explain it."
(RT, Phoenix Park, Year 11 class)

Modelling good practice in detail
Phoenix Park teachers encouraged students to communicate their mathematics thinking to others in the class in some detail. They also provided opportunities for students to discuss useful methods of investigation.

In one lesson, the teacher offered an explicit focus on useful strategies the students could employ in their mathematics explorations. She posed the following question: "If someone new came into class and asked you what makes a good piece of work, what would you say?"
When the students offered suggestions such as, "have an aim", "draw a plan", "write about patterns", she pursued them with further questions:
"What does that mean? Why is a plan important? What does a good plan look like? How do we record patterns?"
The students struggled over their explanations, but stayed engrossed in the discussion for some time. She then gave them a task to design a poster describing the different features of 'good work'.

Phoenix Park teachers showed great confidence in their teaching methods. Readers can find a case study which explores the link between teacher confidence (case study 3), effective methods of teaching and pupil progress in mathematics at the end of this RoM.

How did the 'traditional' school handle differentiation?
Amber Hill provided differentiation for students in several ways. In Year 9, the students were placed in eight different sets, based on ability. The teachers believed that the reduced spread of ability within each class enabled them to teach methods and procedures to the whole group together. As one teacher said:

"It's easier to pitch your lesson... to teach them all together from the front, as a class."
(EH, mathematics teacher, Amber Hill)

The pace of work varied between each set. The staff expected top set students to cover a greater quantity of work and a broader curriculum than other students. Teachers at Amber Hill generally pitched their lessons to the middle of the group. However, the study found that almost all the students seemed to find some parts of lessons either too slow or too fast. The pace of working at which students felt comfortable was determined their prior experience, individual preferences or even their feelings on the day, as well as by the difficulty of the topics.

Within any given set, the work was differentiated by outcome, insofar as quicker students would complete more questions and better students would get a higher proportion of the questions right. In addition, Amber Hill teachers tried to differentiate their questioning by asking more open questions of students whom they felt understood the work and asking students who seemed to be struggling tightly defined questions that required little thought to answer. Sometimes, these questions were simple calculations or merely required a choice between two possible answers. The staff at Amber Hill generally believed it was necessary to provide less able students with a greater degree of structure to help them reach the answers. They had low expectations of many students and thought closed questions would help them. Some teachers gave non-verbal clues to help students reach the correct answer. Amber Hill teachers also aimed to differentiate by support. As students worked from textbooks, they moved around the classroom, spending time with those who seemed to be struggling.

This pattern of differentiation is not unusual. Nevertheless, the study showed that it failed to meet the needs of a number of students. This is explored further on the following page.

Ability grouping does not necessitate whole class teaching at a fixed pace, but this practice is fairly common in British schools. Readers may be interested to explore a study of whole class interactive teaching (case study 4) which identifies aspects of whole class mathematics teaching that make learning more effective.

Did setting have disadvantages for students?
For many of the students, setting meant:

- disaffection related to the restricted opportunities for lower sets
- perceived discrimination in setting decisions
- anxiety, a lack of understanding and underachievement, especially for girls, created by the fast pace of high set environments.

Readers may be interested in looking at an earlier RfT on setting and ability grouping.

Restricted opportunities

The sets in which students were placed determined both the level of examination for which they would be entered and the maximum grade they could achieve. Grades of A*, A and B were only available to students in Sets 1, 2 and 3 and grades for the lowest sets were restricted further. Older students knew this and many expressed anger and disappointment at this apparent unfairness. The following comment was typical:

"You're gonna get an E and there's nothing you can do about it and you feel like...what's the point in trying? What's the difference between and E and a U?"
(Simon, Amber Hill, Year 11, Set 7)

It was possible, in theory, for students to move between sets, but this rarely happened. Students were assigned to sets by the age of 13, so these early setting decisions had lasting repercussions.

Allocation to "the wrong set"

Some students mentioned in interviews that they felt they were in the wrong set. The study found some evidence that this might be true. It compared students' eventual GCSE grades against their ability scores (as measured by the NFER-Nelson mathematics test taken at the end of Year 8). The results showed that some students at Amber Park made greater progress than might be expected and others did less well than early measures of their ability might predict. The former students were more likely to be middle class and the latter to be from lower social classes.

Setting decisions at Amber Hill were based on students' NFER-Nelson scores and other measures of mathematics attainment from Years 7 and 8. Nonetheless, when the researcher matched students for ability, she found those of a lower social class were significantly more likely to appear in a lower set. The study concluded that the disproportionate allocation of working class students to lower sets would have restricted the achievement of working class students.

Lesson pace and its link to top set underachievement

Amber Hill teachers felt compelled to move quickly through the work. This meant that they did not spend time on pupils who could not provide correct answers. They worked through demonstrations on the board, asking students questions and moving swiftly around the class until they heard the right answers. The syllabus for higher sets was more extensive, so the higher the set the students were in, the more likely they were to experience this fast, intense mathematical experience.

The study uncovered unexpectedly high levels of disaffection amongst top set Amber Hill students. They were consistently more likely than lower set students to state that they never enjoyed mathematics and that they were always anxious in lessons. Despite being placed in the top set on the grounds of ability, no girls and only two boys described themselves as good at mathematics. More than half of the top set students complained that the pace of the lesson was too fast for them to be able to understand the work.

The top set experience at Amber Hill seemed to have a particularly detrimental effect on girls. A comparison of early indicators of mathematical ability with later measures of attainment identified 15 students in the top set who were underachieving, of whom 11 were girls. Most of them were easy to identify in lessons. Several sat together and looked lost, confused and unhappy in lessons. They completed hardly any work. The two girls who attained the highest and second highest NFER-Nelson marks in the school in Y8 attained only a Grade E at GCSE. They were both clear that the pace in the class was too fast to enable them to understand. In this set, of 16 girls and 17 boys:

- boys attained 14 of the 19 A to C grades

- girls attained 11 of the 14 D and E grades.

These findings add to evidence from elsewhere that has shown that the greatest remaining gap in mathematics performance between boys and girls occurs amongst the top five per cent of students.

What did the author conclude from her three-year study?
One conclusion of the study was that the students at the two schools developed different types of mathematics knowledge, rather than simply acquiring more or less knowledge.

The Amber Hill teachers were committed and competent mathematics specialists. Nevertheless, the model of teaching and learning that they followed placed some students at a disadvantage. These teachers did not see any difference between a clear transmission of knowledge and student understanding. They seemed to consider that learning and understanding were the same. They gave students closed and structured pieces of mathematical knowledge to learn at a rote level and did not offer them opportunities to think about, use or discuss mathematics. The researcher proposed that most of the problems of misunderstanding, disaffection and underachievement experienced by the Amber Hill students came from this model of 'knowledge transmission'.

The students in Phoenix Park were required to interpret mathematical situations, choose methods and adapt them and solve problems in a way that mirrored mathematics in the real world. Whereas Amber Hill students felt constrained to search for correct answers and the 'right way' of approaching a problem, Phoenix Park students were allowed to try out different options. They were not given and did not depend upon the non-mathematical cues which the Amber Hill students used to help them. When Phoenix Park students encountered difficulties, the teachers did not tell them what to do. They asked them questions that encouraged them to think and make connections between the problems on which they were working and the mathematical methods they had learned.

The students in the more 'traditional' school appeared to be passive learners. They depended heavily on their teachers, becoming adept at finding cues as to how to proceed from the way in which their teacher presented information or asked questions. They learned to repeat procedures and follow rules and were rarely required, or given the time, to think deeply about their mathematics. Whilst the Amber Hill teachers stated their belief that students should be given opportunities to find their own way to solutions, the evidence showed that they did not do so.

The Amber Hill teachers tried to help students by offering them sets of rules or 'handy hints' that were easy to learn. The researcher suggested there was a mismatch between what the students and the teachers gained from these rules. The teachers understood the mathematical context, so to them, rules were meaningful and useful. The students lacked this broad understanding, so to them, the rules were difficult to use and were merely abstract concepts to be remembered.

Readers may wish to read a case study that explored student misconceptions about rules in algebra (case study 5) and how careful attention to the precise use of mathematical language helped them to a deeper understanding.

How was the study designed and undertaken?
This study was conducted in England between 1992 and1995, at a time of change. The national curriculum in England and Wales had recently been introduced and changes were being made to the mathematics GCSE examination. OFSTED school inspections began the year after the study ended.

The study tracked a cohort of students in two schools in England over three years. During this period the cohort progressed through Year 9 (composed of 305 13-14 year olds), Year 10 (268 14-15 year olds) and Year 11 (290 15-16 year olds). One of the schools chosen was a secondary school, taking students from Year 7 onwards, and the other was a high school, taking Year 9 and above. This may have affected the way in which the curriculum and teaching groups were organised. However, the schools chosen were similar in terms of socio economic composition and ability of pupils and both served working class areas. The students at the schools were similar in terms of gender, ethnicity and ability. They had received similar experiences of mathematics before the start of the study.

The researcher established a baseline of attainment at the beginning of the study and took various measures throughout the study and at the end. She investigated students' attitudes towards mathematics teaching and learning and teacher attitudes and beliefs.

The researcher collected a wealth of data. This included a number of tests and assessments of student attainment in mathematics:

- standardised NFER-Nelson test results for all students taken at the end of Year 8
- short contextualised questions including knowledge and use of mathematics in different contexts - this type of test was administered twice to all students: once at the start of Year 9 to provide a baseline measure of students' performance at the start of the study; once at the end of Year 10 to provide a comparison with performance on the similar Year 9 tests
- researcher designed tests of mathematical problem solving from a sample of pupils in Year 9 and Year 10 and associated prior tests of relevant mathematical skills
- long-term learning tests: the same test on a piece of school work administered three times - once before studying the topic, once immediately after studying the topic and once six months after finishing the topic - these tests were performed during Year 10 for two groups in each school (61 students)
- GCSE mathematics results for Year 11, including a detailed analysis of the completed papers that the researcher was given special permission to undertake.

The study also collected a great deal of information from both students and teachers on attitudes and behaviour, including:

- pupil questionnaires administered to all students in the study cohort in the second and eighth terms of the study
- pupil questionnaires administered to all 653 students in Years 8,9 and 10 in both schools in the fifth term of the study
- interviews in the sixth term of the study with 16 Year 10 pupils from each school
- interviews in the eighth term of the study with 24 Year 11 pupils from one school and 20 Year 11 pupils from the other
- interviews in the first term of the study with four teachers from the one school and three from the other
- interviews in the eighth term of the study with three teachers from each school
- about 100 lesson observations in each school during the course of the three years.

What are the implications for practitioners?

The study has a number of implications for practitioners about the nature of learning in mathematics, the different learning preferences of girls and boys, effective methods of differentiation, the difficulties of balancing requirements for curriculum coverage with depth of understanding and the dangers of providing too much structure or too little scaffolding in attempts to support students. Inevitably, both schools had strengths and weaknesses, so practitioners will wish to use their discernment in choosing which practices to adopt.

Teachers may like to consider the following questions:

- Students may appear to be productive when in fact they are working mechanically, rather than actively thinking. What could you do to increase opportunities for students to work through problems that require thought and to reward students who demonstrate careful mathematical thinking?
- Would your students find it helpful to have more opportunities to discuss with each other what they can do if they get stuck? Could they record the strategies they discover in a way that would benefit the class, for example, by making a poster? (See page 13.)
- When introducing open-ended activities, what do you do to prompt students to discuss the problem fully and to encourage them to record it in their own mathematical language? Could such discussion be used to make sure pupils know how they might make a start?
- Would you find it helpful to share ideas with colleagues about strategies for encouraging students to think out loud and to communicate their mathematical thinking to one another?
- How much do your students vary in terms of the pace at which they work and how quickly they grasp new concepts? Are you aware of any students who are swift at arithmetic but slower to understand new ideas, or quick to understand concepts but who calculate slowly?
- What are your students' views about mathematics lessons? Would finding out make a difference to your teaching?

School leaders may like to consider the following:

- Are professional development activities in place (such as peer observation and feedback) to help teachers to be more aware of their preferred teaching techniques and the effects these have on their students?
- Encouraging students to undertake a greater quantity of open-ended problem solving in mathematics is likely to reduce the apparent quantity of work completed and to be more difficult to assess and monitor. Would it be helpful to hold a workshop to communicate the benefits of this type of work to parents and other stakeholders?
- If your school uses setting by ability in mathematics, are you confident that setting decisions are independent of socio-economic factors and behavioural issues? How could you increase the chances of success in public examinations for students in lower sets? For example, could your school offer an alternative accreditation system (such as GNVQ) or run catch-up sessions?
- The specific characteristics of top set mathematics described in the study are not a necessary aspect of ability grouping. Would it be useful for staff to discuss effective strategies for differentiation within sets?

Your Feedback
Have you found this study to be useful? Have you used any aspect of this research in your own classroom teaching practice? We would like to hear your feedback on this study. To share your views with us email: research@gtce.org.uk
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Case studies

We have selected the following case studies to illustrate different aspects of improving teaching and learning in mathematics. All the case studies were drawn from classroom-based research conducted by teachers. With the exception of case study 4, the studies took place with secondary school students. Readers with a particular interest in the teaching of primary school mathematics will find links to studies from primary and infant schools in the section on Further Reading.

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Developing a need for algebra

Case study 1 - Developing a need for algebra

This study has been chosen as it offers an example of how a mathematics classroom was set up to enable the students to learn algebra more effectively via an open-ended, investigative style of teaching and learning.

The study investigated what aspects of classroom climate and organisation helped students to develop a sense of the power of algebra and to use it themselves without prompting. The study looked at one class of twenty-seven 11 and 12 year-old students and took place over the period of their first term in secondary school. The class was of mixed ability, with a high proportion of less able students.

The teacher researcher was most interested in getting students to use algebra in a way that involved working backwards, problem-solving, explaining and justifying, awareness of the constraints of the problem situation and awareness of mathematical structure. He wanted to promote a particular classroom culture that enabled students to become mathematicians.

In order to do this, the researcher:

- explicitly shared the aim with students
- commented on and highlighted mathematical behaviour by students
- asked students to write, both when solving mathematics problems and when reflecting on what they had learnt
- put self-checking mechanisms in place so that he was not the sole arbitrator whether students were right or wrong
- set up common boards such as a 'questions board' and a 'theories board' to stimulate class discussion and so students could review one another's work and ideas
- constantly encouraged the students to ask their own questions
- asked open questions that allowed students to generate their own work.

He used video to monitor his teaching strategies and reflect on his practice.

How was the classroom culture established?
The study highlighted four strategies for building a classroom climate that encouraged students to ask their own questions and hence to find a need for algebra.

Becoming a mathematician
In the first lesson of the year the teacher told the students that this year was about 'becoming a mathematician' and that this meant:

- thinking for yourself
- noticing what you are doing
- asking why things work
- being organised
- looking for patterns.

This allowed the students to become more aware of what they did in mathematics lessons and helped them and the teacher to question whether their actions were mathematical or not.

Highlighting examples of mathematical behaviour
The teacher aimed to comment on behaviour that showed someone 'being a mathematician' in every lesson. For example, when a student approached a problem in a systematic manner he said, "That's an excellent example of getting organised, which is part of thinking mathematically". The students began to use such phrases as 'getting organised' and 'asking why' in their writing and in talking about what they were doing. Students offered their own examples of being a mathematician, such as 'it's okay to make mistakes' and 'you share your problems with other people'.
The choice of activities and teaching

The teaching was open-ended in order to give students the opportunity to experience 'getting organised'. The teacher chose classroom activities that were accessible to all and could be extended through asking questions. He carefully determined when the moment was right to introduce new skills to the pupils. The students practised the new skills within the context in which they arose. The common boards on which students could pin up things for comment (e.g. to get others to check, to ask a question) or to record results were vital mechanisms for students to check their own and others' work. It became natural for students to ask their own questions in the context of classifying such information.

Emphasising students' writing

The teacher encouraged students to write down what they were doing and what they noticed or found out. This was established as being part of 'becoming a mathematician' as it helped the students to make links in their learning and promoted their awareness of what they were doing.

Did the students develop a need and ability to use algebra?
The evidence from this project showed that students did develop a need for algebra within the culture of the Year 7 classroom. Before arriving at secondary school, the students had received little or no exposure to algebra or to ideas of proof.

By the end of one term, students from across the whole attainment range displayed an appreciation of two different uses of algebra:

- to prove a result which they believed to be true
- to show 'the workings' of a problem i.e. the use of algebra to express a rule.

In addition, several students could use algebraic proof to answer a question which they themselves had posed within a problem situation. Over half the students showed evidence of using algebra to express their ideas.

The study found that:

- it was not necessary to drill the students in techniques
- at the end of 15 weeks, over half of the students in the class had moved from never having met algebra before to being able to use algebraic symbols to express their own ideas
- students asked with increasing regularity in problem-solving situations: 'Can we do this for N?'
- students showed understanding of the meaning of algebraic statements in different contexts and appreciated when the statements were useful or not within those contexts
- students developed skill in algebraic manipulations as they made sense of algebraic ideas within the context of the activities chosen
- when the first coursework task was presented to these students they were able to structure their own approach to the problem in a way similar to that required in GCSE coursework.

Reference

Coles, A. (1999) Developing a need for algebra Teacher Research Grant summary, 3rd year 1998-99.

Cross curricular work in mathematics and science

We chose this study because it illustrates how several schools used data from real situations found in science to develop skills in mathematical analysis and modelling.

The study took place in one primary and three secondary schools, involving pupils in Years 6, 9 and 10. The teachers worked collaboratively to plan cross curricular lessons in mathematics and other subjects, especially science. They tried out team teaching and observed one another to develop their understanding of what was happening in each subject.

The students used graphing calculators that were inexpensive and could be carried around easily to explore a variety of situations:

- junior school pupils linked up temperature and light probes and plotted temperature and light intensity over the course of a day to investigate links between the two
- senior school students compared the efficiency of washing powders by measuring the amount of light that passed through dirty and cleaned fabrics

- one school investigated the factors affecting the height to which a ball bounced
- other students used a motion sensor linked to a graphical calculator to explore how their motion produced graphs on the screen.

The projects helped to deepen understanding and also allowed student to work at different levels. Many students find it difficult to interpret distance/time graphs and speed/distance graphs. The students who struggled to interpret graphs were able to gain a greater understanding of graphs because they could see immediately how their motion was turned into a graph on screen. Meanwhile, high attaining students were challenged by having to find an equation that fitted the motion. They were able to measure the value of acceleration due to gravity. They understood that the gradient of the graph represented the speed of the moving object. Students used the sensor to log the speed of passing cars and were delighted to discover that when they pointed a strange gadget at a car, it almost invariably slowed down!

Cross curricular work later spread to include:

- field work in geography on the flow rate of water in a stream
- investigations in Physical Education on how exercise affects heart rate.

Positive outcomes of the project included:

- Cross curricular planning allowed the exchange of ideas between staff and helped them to expand their knowledge of their own and other subjects.
- Mathematics teachers were able to exploit real data from other subjects to enrich different areas of the mathematics curriculum.
- The two subjects were used to support one another so that time was not wasted in repetition.
- The use of real data and instant feedback gave pupils a greater understanding of the diagrams and formulae involved in mathematics and science.
- The technology allowed quick and easy data capture and the students could retake readings if they looked dubious.
- The students engaged in higher order thinking skills such as making predictions, analysing data and modelling data with equations.
- The students were able to work more independently.
- The projects offered opportunities for effective differentiation.
- Students enjoyed the work and became more interested in mathematics and science.
- In an informal survey, over half the students stated that they thought cooperative group work helped their own and others' learning and that it was more useful and important than they had previously believed.

Reference

Ransom, P. (1999) Cross-curricular work involving mathematics using hand-held technology Teacher Research Grant summary 3rd year,1998/99.

Teacher confidence in teaching data handling

We have chosen this study because it shows how focussed support was successfully used to deepen teachers' understanding of, and to raise their confidence in, teaching data handling. It also shows a subsequent gain in the attainment of their pupils in this area.

The study took place in two schools for pupils aged 4 to 9. It set out to find whether there was a relationship between low levels of teacher confidence in the teaching of data handling skills and pupil achievement in the same area. It found a link between the two. When staff confidence and knowledge was improved by the provision of focussed support, including observation and discussion of lessons and joint lesson planning, pupil achievement also increased significantly.

The teacher researchers analysed QCA test results in mathematics. They found that pupils made most mistakes in the data handling questions. They gave a preliminary questionnaire to all staff in the two schools and found that several colleagues felt they lacked confidence and needed support in aspects of mathematics. The researchers chose the six teachers who showed the lowest confidence in teaching mathematics, especially data handling, and offered them a carefully tailored programme of support.

The researchers found that teachers lacking in confidence tended to rely on a published mathematics scheme to teach data handling. They felt that this limited children's achievement in this area. They helped the teachers in the study to develop a bank of data handling ideas that could be used to offer the children meaningful practical experiences. They observed lessons, discussed these with staff and jointly planned lessons as part of their support programme.

During the course of the study, all staff received INSET support on mathematics teaching, which included a significant amount on probability. The teaching of probability was regarded as being particularly difficult.

What difference did this make?

At the end of the study, the researchers found that the teachers who were given focussed support reported a greater increase in confidence in teaching data handling skills than the rest of the staff. The teachers involved in the study:

- all reported an increase in confidence
- reported an average level of increase of +4.7, compared with an average level of increase in the remaining nineteen teachers of +3.4
- showed an increased range of questioning and ability to ask both closed and open questions.

During the same period, the children's Year 4 QCA test scores showed a marked improvement. Their results in data handling improved, on average, by 47 per cent in one school and by 35 per cent in the other. In addition, in class, the children showed:

- increased task focus
- better responses to questions
- a deeper understanding of the issues involved in collecting, representing and analysing data.

The researchers confirmed their observations in class during conversations with the children.

The study also found that, despite the whole staff INSET training, few children answered the probability question in their QCA tests correctly. The researchers concluded that the INSET training was not as effective in improving teaching as the more classroom focussed support programme.

Reference

Price, R. & Raiker, A. 1999 Teacher confidence and teaching and learning in data handling Teacher Research Grant summary, 3rd year 1998-99.

Whole class interactive teaching in mathematics

We chose this case study because it illustrates how whole class teaching can be used effectively to improve the quality of students' learning in mathematics.

The study brought together teachers of mathematics at Key Stage 3 (11-14 years) from six secondary schools in the north of England. They shared an interest in trying out methods of teaching mathematics which had led to high student attainment in Hungary. They met once every half term to work with members of the mathematics department at Manchester Metropolitan University (MMU). The group exchanged ideas with visiting Hungarian educators and some visited Hungary to observe lessons there. The Hungarian classrooms showed high levels of whole class teaching, classroom discussion and a willingness amongst students to discuss their mistakes or difficulties openly.

The British teachers visited colleagues' lessons, watched recorded lessons on video and discussed lesson descriptions to agree on a shared vocabulary they could use to describe features of lessons. They tried out new ideas in class and used video to record what happened. They discussed and analysed the videos as a group to refine their theories as to what happened and why. Having identified the effective structures, strategies and activities of whole class interactive teaching in mathematics, they put them into practice in further lessons. This, in turn, provided data for further refinement at future workshops.

Key findings from this case study

The study found that effective whole class interactive teaching involved:

- asking students to make their beliefs and difficulties public
- focusing in depth on a small number of significant problems
- focusing on key mathematical ideas and misconceptions
- promoting high levels of articulation in students of all abilities
- making changes to teaching strategies.

Making beliefs and difficulties explicit
Hungarian students regularly came to the front and demonstrated their mathematics to the rest of the class on the board. The teacher researchers did the same but were concerned about students' sensitivities to this exposure. They made a distinction between the students and their mathematics and emphasised that the mathematics was the focus for discussion. This encouraged co-operation and mutual support, so students felt able to expose and explain their ideas and difficulties.
Focusing in depth on a few significant problems and on key mathematical ideas and misconceptions

The group found that explanation followed by exercises made students look busy but did not help them think about their mathematics. Learning was more effective when:

- students saw the tasks provided as interesting and worth resolving
- the task was clear and the problem easy to access, even the mathematics
- concept or content was challenging and hidden in the task
- students discussed a small number of problems in depth
- teachers focused on issues and concepts rather than just techniques
- students were allowed to fall into traps and then explored their partial understandings, beliefs, feelings, instincts and misconceptions.

Whole class interactive teaching helped students of all abilities
The researchers found that all students, including those in lower ability groups, were willing and able to discuss their mathematics openly. A supportive environment helped everyone to discuss their own mathematical ideas and to produce explanations and justifications of their work.

Changes in teaching strategies
The researchers found the following teaching strategies helpful:

- valuing student contributions without judging them
- recognising when to facilitate discussion and when to intervene
- developing high level questioning and prompting skills
- insisting on the use of correct mathematical language.

Teachers needed to develop:

- an awareness of students' beliefs and misconceptions
- an awareness of key mathematical ideas over and above the learning of algorithms
- an awareness of tendencies in themselves, for example, to jump in to correct students, take over explanations or explain away difficulties
- strategies for designing problems which prompted students' awareness of key mathematical ideas. Such problems needed to tap into students' beliefs and be rich enough to provide a significant link to the mathematics.

The nature of lesson preparation changed so that teachers could be more responsive and increase their repertoire of activities and questions to help students gain insights into the topic being studied.

What were the benefits to teachers and students?

- The teachers who took part felt they gained a lot from the experience and believed, on the basis of video evidence, that their practice had improved.
- They were more alert to opportunities that emerged from student responses.
- They used class discussions much more to deepen understanding than to explain procedures.
- The participants found video helpful for individuals to see themselves clearly and for the group to work on one another's ideas.

The teachers judged the effectiveness of their lessons on the students' achievement. They found that the students:

- became more effective learners and communicators than before
- became more articulate in explaining and justifying their mathematical thinking and decision-making
- could cope with harder mathematical problems than would be expected for their age.

Reference

Harrington, A. (1999) Whole class interactive teaching in mathematics Teacher Research Grant summary, 3rd year 1998-99.

Language issues in the teaching and learning of algebra

We have chosen this case study because it identifies some of the difficulties students experience and the misconceptions they may adopt in algebra if the subject is taught in a way that emphasises routine skills rather than understanding and if explicit attention is not paid to the careful use of technical language.

The research was carried out by a teacher in a Kent school with 55 students aged 14 to 16 and two teachers at the same school. The researcher observed mathematics lessons, interviewed the students and teachers, and tested the students' ability in algebra, problem solving and reading.

The study found that:

- students with low reading ages and poor problem solving skills could still become competent at algebra
- misconceptions arose when students did not understand the complex language used by the teachers
- students found it difficult to describe and discuss algebra and generally did not use the language used by their teachers.

Why do students find algebra hard?
The study found no direct relationships between students' scores on the algebra test and either their reading age or their score on a problem solving test. The researcher was therefore cautious about the assumption that, if a student was generally academically weak, they would also struggle with algebra. The study tried to determine what particular issues caused difficulties for the students in their understanding of algebra.

The study found that many students:

- assumed that they always had to find an unknown value when doing algebra
- believed that different letters could not have the same numeric value
- thought the alphabetical order of the letters was important
- found using algebraic terms and numbers simultaneously was confusing
- found it difficult to explain what terms mean
- could not explain the different uses of algebra
- found it difficult to grasp and work with abstract concepts.

They made comments such as:

"I suppose it must be useful but I don't know why letters are used so much."

Students seemed to form simplistic notions quite early on and did not change these in response to being taught new material. When faced with complex problems, students did not use the strategies they had been taught. They adopted a common sense approach but struggled because of the misconceptions highlighted above. What new teaching strategies might be useful?

In the interviews, the teachers reflected that, although the choice of language used when teaching algebra was crucial to students' understanding, they had not made a conscious effort to teach it. They determined to be much more explicit about the teaching, revision and use of appropriate language.

They stated that algebra was the area of mathematics that was most difficult for them to teach. They felt that they needed to prepare more for these lessons. The teachers also felt that some student misunderstandings resulted from an over-emphasis on routine skills at the expense of developing students' conceptual understanding.

The study suggested that:

- teachers might find it helpful to work together to share their experiences and to think through their strategies carefully
- diagrams could be useful to help students' understanding
- key vocabulary should be explicitly taught
- teachers should avoid using codes (a=1, b=2 and so on) when introducing algebra
- that students tended to find it easier to use algebra to describe a procedure than to develop mathematical structure.

Reference

Wearden, G. (1998) Language Issues in the Teaching and Learning of School Algebra. Teacher Research Grant summary 2nd year,1997/98.
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Further reading

What else might I enjoy reading?

Boaler, J. (1997) *Experiencing school mathematics: teaching styles, sex and setting*. Buckingham: Oxford University Press

Orton, A. & Frobisher, L. (1996) *Insights into learning mathematics*. London: Cassell

Cockcroft, W. H. (1982) *Mathematics counts: report of the inquiry into the teaching of mathematics in schools*. London: HSMO.

Boaler, J. (1997) 'Open and closed mathematics approaches: student experiences and understandings'. *Journal for Research in Mathematics Education*, 29 (1) pp. 41-62.

Lubienski, S. (2000) 'Problem solving as a means towards mathematics for all: an exploratory look through the class lens'. *Journal for Research in Mathematics Education*, 31(4), pp.225-248.

Boaler, J. (1997) 'Setting, social class and the survival of the quickest'. *British Educational Research Journal*, 23 (5) pp. 575-595.

Straker, A. (1993) *Talking points in mathematics*. Cambridge: Cambridge University Press

Wiliam, D. and Bartholomew, H., (2001) *The influence of ability-grouping practices on student achievement in mathematics*. Paper presented at the 27th annual conference of the British Educational Research Association, University of Leeds, September 2001.

Barnes, D. (1992) 'The role of talking in learning'. In K. Norman (ed.) *Thinking voices: the work of the National Oracy Project*. London: Hodder and Stoughton

Boaler, J., (1997) 'When even the winners are losers: evaluating the experience of 'top set' students'. *Journal of Curriculum Studies*, 29 (2) pp. 165-182

Where can I find out more online?

You may also like to read our earlier RfT summary about effective teachers of numeracy at primary level:
Association for Teachers of Mathematics
www.atm.org.uk

DfES National Numeracy Strategy web pages
www.standards.dfes.gov.uk/numeracy/

Mathematical Association
www.m-a.org.uk

The Qualifications and Curriculum Development Agency
Has links to the National Curriculum website and the National Numeracy Strategy, as well as many other resources for teachers and parents.
http://www.qcda.gov.uk/

You may like to read our earlier RfT about ability grouping in education.
http://www.gtce.org.uk/teachers/rft/group0504/

Related research

www.ncetm.org.uk/enquiry/9251

Effective pedagogy in mathematics Best Evidence Synthesis:
www.educationcounts.govt.nz/__data/assets/pdf_file/0007/7693/BES_Maths07_Complete.pdf

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Appraisal

Boaler, J. (2002) Experiencing School Mathematics, (Revised Edition) New Jersey: Lawrence Erlbaum Associates

Robustness
This book reports on a study conducted in England between 1992 and1995 that was first published in 1997 in the UK, and in 2002 in the US.

The study is an evaluation of two different naturally occurring approaches to teaching mathematics, characterised respectively as traditional and progressive (called 'reform' in the US). The study took place during a period of change in England including the introduction of the mathematics national curriculum and changes to the mathematics GCSE examination. The researcher reviewed research literature about mathematics that covered studies of how individuals used mathematics outside the classroom in a range of real-life contexts. The researcher also considered studies showing that students who were taught using open or activity approaches performed better on tests.

The research tracked a cohort of students in two secondary schools in England over three years. During this period the cohort progressed through Year 9 (n=305 13-14 year olds), Year 10 (n=268 14-15 year olds) and Year 11 (n=290 15-16 year olds). The two schools were similar in terms of sex, ethnicity, socio-economic composition, and abilities of students. The schools were chosen because they had two different philosophies - one (traditional) where the mathematics department mainly used a text book approach with pupils set in ability groups, and the other (progressive) in which mathematics was taught to mixed ability classes using open ended activities differentiated by outcomes.

A good range of data was collected. The researcher used standardised NFER tests from Year 8, pupil questionnaires and tests administered for each year of the study and GCSE mathematics results for Y11 in order to assess changes in attainment, and in students' attitudes towards mathematics teaching and learning over the period of the study. These data were complemented by a number of other data sources, which together provided a rich description of mathematics teaching and learning in the schools. They included 100 lesson observations, interviews with 32 students in Y10 and 44 pupils in Y11; interviews with thirteen teachers of Y8 and Y11 students and researcher designed tests of mathematical problem solving from a sample of students in Y9 and Y10.

Relevance
The findings relating to open-ended approaches which were linked with improvements in students' understanding, in their confidence about using mathematics and in their performance in GCSE examination results should prove interesting to schools and teachers of mathematics. Whilst focused on the 14-16 years age range, the messages from the research are also relevant to teachers in other phases. Although the study reported on students' experiences in the mid 1990s, and some practitioners today may find the context of the study dated, the findings should help to inform current thinking about the curriculum and approaches to teaching and learning.

Applicability and accessibility
There was a clear focus in the study on teaching and learning that appeared to have influenced learning gains

in mathematics. The findings are illustrated with a range of material relating to mathematics teaching and learning in contexts familiar to teachers of mathematics in mainstream schools in England. Teachers will find it helpful to consider their own teaching strategies and activities in the light of those described in the study including the use of open-ended approaches, setting problems in the context of real-life situations, allowing students to work at their own pace, the preferred teaching and learning styles as expressed by boys and girls, and the impact of different ways of grouping students. The findings relating to underachievement should prove useful to schools and policy-makers.

Writing

In revising and updating this study for an audience of US practitioners the author has had to explain terminology used in England and Wales, but this does not detract too much from the essence of the report. The study is written in a way that is likely to engage many readers and provides a great deal of thought provoking evidence and analysis that could help teachers to reflect on their own practice today.

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