Research for Teachers

Effective teachers of numeracy

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What factors help to make some teachers of numeracy more effective than others?

Askew, M., M. Brown, V. Rhodes, D. Johnson and D. Wiliam. *Effective Teachers of Numeracy*. London: King's College London, 1997.

The study featured in this TLA research summary set out to identify and describe the characteristics of effective teachers of numeracy. It took place before and during the introduction of the National Numeracy Strategy.

The researchers were asked to feed back their findings into the development work for the strategy. They explored teachers' beliefs, knowledge and understanding as well as their teaching practice. They found that developing a rich network of connections between different mathematical ideas and using pupils' reasoning to help establish these were most likely to result in greater pupil achievement and identified a range of other factors that contributed to effective numeracy teaching.

The study helps demonstrate some important links between teacher understandings, teaching and learning that may help teachers to take a fresh look at their own practice in the light of their knowledge and beliefs about numeracy.The case studies, which illustrate teaching and learning in classroom contexts, are not taken from the main study itself, but are independent studies from a number of other schools.

Back to top

Overview

Why is the issue important?

In the current climate of concern about national standards, research that explores the characteristics of

effective numeracy teaching is highly topical. It is also important that teachers are aware of some of the thinking behind the National Numeracy Strategy.

What did the research show?
The classes of teachers with a strongly 'connectionist' orientation tended to make greater gains over the six-month period of the study than the classes of teachers with strongly discovery or transmission orientations. Mathematics-focused professional development helped teachers become more effective in putting over concepts to their pupils.

How was this achieved?
Connectionist teachers:

- connected different areas of mathematics and different ideas in the same area of mathematics
- believed it was important that pupils knew different calculation methods and were able to choose the most efficient method for the problem in hand
- emphasised the development of mental skills
- probed pupils' reasoning to help establish and emphasise connections and address any misconceptions
- encouraged purposeful discussion in whole classes, small groups and with individual pupils
- used assessment to inform planning and teaching.

In so doing, they helped pupils to acquire and apply knowledge of numbers and skills in a variety of contexts.

How was the research designed to be trustworthy?
The study involved ninety teachers of Years 2-6. The teachers' practice was analysed and their beliefs, knowledge and understanding explored through:

- observation of 84 lessons
- a questionnaire administered to all the teachers
- interviews with six headteachers
- interviews with 54 teachers.

Pupil tests, designed to measure gains in numeracy, were administered at the beginning and end of the six-month period of the research. The gains in mean class scores were used to classify the teachers into those who were highly effective, effective and moderately effective.

What are the implications?
The study showed the importance of teachers:

- having a 'connectionist' rather than 'transmission' or 'discovery' orientation
- developing pupils' mathematical thinking, through, for example, asking pupils to articulate their reasoning verbally or in writing, and discuss and evaluate their methods
- using formative assessment as a means of identifying what pupils have learned, where they are struggling and what needs to be covered next
- working collaboratively with colleagues and sharing their approaches to teaching.

What do the case studies illustrate?
The case studies show:

- the way in which a thinking skills strategy helped pupils learn mental calculation strategies
- how teachers used interactive methods in whole-class teaching to encourage pupil-teacher and pupil-pupil dialogue in secondary mathematics lessons
- how teachers in one primary school developed children's skills in explaining their reasoning
- ways teachers successfully built on children's errors to help them reach a better understanding of mathematical methods.

Back to top

Study

What was the aim of the research and how might it help practitioners?
The study was set up to identify:

- key factors which enable teachers to put effective teaching of numeracy into practice in primary schools
- strategies that would enable those factors to be more widely applied in numeracy teaching.

By collecting and analysing data about teachers' beliefs, knowledge, understanding and practice, the researchers' intention was to identify what it was that made some teachers of numeracy more effective than others. (A definition of effective numeracy teachers is given in the next section).

Evidence in the form of pupils' class gains in tests offered the researchers the opportunity not only to triangulate their data but also to assess the extent to which pupils in the schools developed mathematically over the period of the research and provided a measure of teacher effectiveness.

The report incorporates an extensive analysis of the factors that had an effect on the practice of the teachers in the study. This offers teachers a helpful framework, which they might use to reflect on their own practice and the beliefs, knowledge and understanding that underpins it.

How was the research designed?
Central to the study was the identification of teachers regarded as effective in teaching numeracy. Firstly, the researchers defined effective numeracy teachers as those who help pupils:

- to acquire and apply knowledge of numbers, number relations and number operations based on the integration of understanding, techniques, strategies and application skills
- to learn how to apply their knowledge and skills in a variety of contexts.

The researchers then considered the possible means of judging teacher effectiveness and suggested the following as potential sources of evidence:

- teacher behaviour
- pupil behaviour
- pupil learning outcomes.

They recognised the inherent difficulties of the first two approaches. In the case of teacher behaviour, there was very little information about what sort of teacher practice leads to effective pupil learning. Indeed the research project itself aimed at providing such evidence. The difficulty in measuring teacher effectiveness in terms of pupil behaviour, on the other hand, lay in the fact that learning is difficult to observe directly and there is a lack of evidence about the kinds of pupil response that are indicators of sustained learning.

The researchers defined 'effective teachers of numeracy' as those who bring about identified learning outcomes. Teacher effectiveness, of course, could only be related to pupil gains at the end of the research. The researchers applied a two-stage approach to sampling effective teachers, comprising:

- identifying a sample of 'focus' schools that appeared to be effective in teaching mathematics
- identifying a sample of teachers, believed by headteachers to be effective in the focus schools.

Data was collected by a number of methods including:

- questionnaires
- classroom observations
- structured and semi-structured interviews
- pupil test results.

What do we know about approaches to teaching numeracy?
What did the research discover about teachers' beliefs? The researchers found three dominant sets of beliefs and approaches to numeracy teaching and learning amongst the teachers:

- connectionist
- transmission
- discovery.

Having identified these broad patterns the researchers used the resulting models to analyse other data about the pupils and teachers involved in the study. The resulting descriptions of the three approaches and their impact on learning are thus rooted in evidence but are also extended to create models. For clarity, we have described these models in their fully developed forms rather than in the partial forms that emerged from the analysis of the data.

The models describe broad bands or orientations, and cover a number of features of teacher behaviour. Of course, any individual teacher may show some characteristics from each of the three models over the course of his/her teaching, although s/he will usually show patterns of work that fall predominantly within one model.

The researchers suggest that the main ways in which the three orientations differ include beliefs about:

- what constitutes numeracy in pupils
- how pupils can learn to become numerate
- how best to teach pupils to become numerate.

How do teachers' beliefs influence teaching and learning?
The researchers observed differences among teachers in their approach to numeracy teaching methods. For example, teachers adopting a mainly transmission approach valued methods that were based on the use of standard procedures and routines. Teachers who adopted mainly discovery methods, advocated a practical approach to problem solving. Those with a connectionist orientation encouraged methods that placed the highest priority on mental methods. They also regarded it as important that pupils were aware of different methods of calculation and were able to choose methods in relation to their effectiveness and efficiency in solving the problem. Those with a transmission approach emphasised paper and pencil methods.

The researchers observed differences in the way teachers treated pupils as learners. Teachers whose approach was broadly transmission or discovery emphasised either teaching or learning although they differed greatly in the emphasis they placed on one or the other. The former placed emphasis on teachers teaching pupils to follow instructions, while the latter regarded individual activity as central to pupils' learning of numeracy.

Teachers with a connectionist orientation emphasised the complementary nature of teaching and learning and valued classroom activity, which involved pupils working together with other pupils and teachers to overcome difficulties and to reach shared understandings.

The researchers also observed that many teachers with a connectionist approach believed that mathematics should not be taught in a fragmented way and that where appropriate pupils should be introduced to some of the complexities of mathematics. Teachers with either of the other orientations were observed to be more cautious in the approaches they adopted. Numeracy content was more likely to be compartmentalised, and pupils judged to be 'ready' before they could go from one idea to another.

Another difference observed by the researchers related to feedback. Whilst teachers with different beliefs about numeracy teaching and learning may seem to act on feedback from pupils in similar ways, the particular beliefs of the teachers can lead to very different outcomes. Those with a transmission orientation were likely to treat pupils' errors as a result of pupil carelessness or lack of attention and simply correct errors rather than make them explicit. Connectionist teachers who believed that the major factor in learning is that pupils engage and struggle with processes, used pupils' errors as a means of engaging with them in order to further their understanding.

To read a case study in which teachers used pupils' errors to create new strategies for learning, see the case study about the Sharp Lane Primary School Project Report.

What did the research find out about effective teaching of numeracy?
A major finding from the research was that those teachers with a strongly connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strongly discovery or transmission orientations.

Another finding was that the connectionist teachers who were highly effective had engaged in extended continuing development (CPD).

The researchers acknowledge that there is no unique description of the effective numeracy teacher. However, they do highlight approaches from their study which, for these 90 teachers and their pupils, appear to contribute to effective numeracy teaching and which include:

- stressing the connectedness of numeracy ideas rather than compartmentalising them
- using pupils' descriptions of their own methods and reasoning as starting points for engaging with numeracy concepts
- an emphasis on enabling pupils to select strategies according to whether they were both effective and efficient (see, for example, the illustration below)
- emphasising the development of mental skills
- ensuring that all pupils are challenged
- encouraging purposeful discussion about choice of strategies
- using assessment to inform planning and teaching.

The researchers suggest that the connectionist approach to problem solving encouraged methods that were both efficient and effective. As an illustration they gave the following example:

"...while 2016 - 1999 can be effectively calculated using a paper and pencil algorithm it is more efficient to work it out mentally."

Further examples of how effective numeracy teachers used these strategies are presented in the following sections.

How did effective numeracy teachers use mental strategies?
All the teachers involved in the research encouraged pupils to have rapid recall of basic number facts. Knowing number bonds was a skill that teachers of all orientations - connectionist, transmission and discovery - saw as important. However, connectionist orientated teachers viewed mental arithmetic as more than this. They regarded number bonds as the starting point for the development of a conscious awareness of connections and relationships with which to underpin mental agility. As one teacher showing a strong connectionist orientation explained:

"I think you've got to know that they are inverse operations those two (addition and subtraction), and that those two (multiplication and division) are linked, because when you are solving problems mentally you are all the time making links between multiplication, division, addition and subtraction... I think mental agility depends on seeing relationships between numbers and being aware of links."

Connectionist teachers also stressed the importance of estimation:

"If you've got a good ability to estimate and to know what a sensible answer is then you're very quick to pick up if something doesn't sound right. And if you can estimate you can get very quick on mental arithmetic and you get very quick on oral skills."

For an illustration of approaches adopted by one school, which was not part of the original study, to improve mental arithmetic strategies used by children, see the case study about an investigation into pupils' strategies for mental mathematics.

How did teachers encourage classroom interactions to help pupils learn?
Lessons presented by teachers identified as highly effective generally involved a lot of task-related discussion between the teacher and pupils and among the pupils themselves. One teacher explained in the following terms:

"If I am honest with myself I probably spend more time talking with them than doing exercises and things like that...because I want them to be able not just to give an answer, I want them to be able to explain the process and what they are doing."

An illustration of encouraging children to explain their methods was provided by this exchange in a Year 2 class:

Teacher asks the children if anyone can add 9 to 36 without using their fingers. She asks a child to explain how they knew the answer was 45. "Well I knew that 36 add 10 is 46 and I took off one." The teacher reiterates the method and another child says that it can be done by taking off one and then adding 10.

To find out about strategies used by teachers in whole class situations to foster classroom interactions, see the case study about teachers' interpretations of effective whole-class interactive teaching in secondary mathematics classrooms.

A Year 1 teacher provided another example of how teachers can foster classroom interactions between children. The children in her class modelled numbers to 100 by putting cubes in two hoops to represent tens and ones respectively, recording and then reading the numbers on a hundred square. The teacher asked the children how the process might be extended to larger numbers. One child suggested adding a third hoop and a 'lively discussion' took place about where it should be placed and the order in which the digits should be recorded.

Teachers who encouraged classroom interactions were also keen to help children with their reasoning. To read how one school developed strategies aimed specifically at improving children's explanation skills, see the case study about developing skills in mathematical explanation.

Which approaches to using and applying numbers helped to challenge pupils' thinking?
Teachers of all three orientations appreciated the importance of pupils being able to apply their computational skills to real-life problems. For many transmission teachers, however, application of knowledge only involved putting what they had already learned into context:

"I will give them just some basic sums set out ready for them similar to the ones we've done today and then problems where they have got to actually extract the information and use what we are using today."

Connectionist orientated teachers, like their colleagues, also emphasised the ability to apply computational skills. But unlike transmission and discoveryorientated teachers, they did not regard it as essential that pupils should have learned a skill before they tried to apply it. One strongly connectionist orientated teacher explained that she believed that pupils benefited from situations in which they did not always have the skills available:

"It is not always a good thing to always be able to do something, because there is no challenge there then. If

you are not always able to do it that is when you start thinking in a different approach and broadening your base."

Another connectionist teacher echoed this view, explaining that he did not hesitate to provide pupils with challenges that they might not succeed at. With one class, he had set up a challenge in which children had to compare two pie charts. Both pie charts showed preferences of populations for different pastimes but one referred to a population of 80 while the other referred to one of 100. Some pupils did not realise that although 40 people in each sample had stated the same preference (for computers) this did not represent the same proportions of each sample. This required them to relate numbers of people both to the proportions shown in the pie chart and to percentages. The teacher noted:

"...although they are very comfortable with the idea of a pie chart, they haven't really got the idea that a pie chart is actually a precise thing..."

The teacher went on to explain that even when pupils got something wrong it provided the teacher with insight into their thinking.

For an example showing how one school investigated mental arithmetic strategies, including one that built on children's errors to create improved strategies, see the case study about Sharp Lane Primary School Project Report.

Connectionist teachers also generally believed that activities should challenge all children not just the most able. As one teacher put it:

"...but I have the same expectations for the children, I always think about it as not so much what the children are doing as what they have the potential to do."

How can classroom assessment be made more useful to teachers and pupils?
When transmission teachers listened to pupils, the researchers suggest, they
were listening for how well the pupils' explanations matched their own rather than engaging in a dialogue. Similarly, such teachers appeared to use pupil performance marks more to check that what had been taught had been learned rather than that the children understood.

Connectionist teachers, on the other hand, used continuous and varied means of assessment to build up detailed profiles at both class and individual student level, as the following comments illustrate:

"So it's all sort of assessment and focus teaching all the time. I assess every day - what activities have gone on and where each group goes next."

"Every piece of work I do, I just keep a sheet like that...and I write my own notes on there if a child has a specific problem. My planning, my search to find the most suitable method of teaching a child... And that just comes from my own experience and my observations and my constant assessment that I use."

"I use continuous assessment from talking to the children, and from listening to what is going on and looking at the work they are doing."

What are the key interacting factors that affect classroom teaching and what effects do they have on teachers?
The researchers analysed effective numeracy teaching in the context of a model that builds on the work of, among others, Bennett, Summers and Askew, 1994; Shulman, 1987. In the model, teacher practice is the most important factor affecting learning outcomes and teacher practice itself is influenced by teachers' beliefs and pedagogic content knowledge (see below).

It is a complex model that also incorporates the effects of pupils' responses on teachers' practices, beliefs and pedagogic content knowledge. The researchers also suggested that teachers' perceptions of pupils' knowledge, understanding and classroom behaviour would feed back to and influence teachers' beliefs,

knowledge and practice.

Teachers' Beliefs

The researchers regard teachers' beliefs as a crucial element influencing teachers' practice. They suggest that these beliefs are based on ideas about:

- what it is to be a numerate pupil
- how best to teach numeracy
- how pupils learn to be numerate.

The researchers stress the importance of the interactivity of the model described above and provide examples to illustrate this. For example, one teacher used the idea of getting pupils to explain their methods to each other as a strategy to control a lively class. The teacher then discovered that this approach was an effective way of helping pupils understand numeracy better. In this case, changes in the teacher's practice had an effect on the teacher's beliefs.

Another teacher explained how her beliefs about pupils' abilities had been challenged and altered through CPD, which involved her in activities with pupils.

What did teachers know about the classroom activities that would support learning?
The researchers found that the importance of a teacher's own subject knowledge was considerably more complex than they had assumed it would be. They found that there was little to distinguish highly effective teachers from the effective and moderately effective teachers in the sample in terms of:

- formal knowledge of mathematical concepts
- qualifications and experience.

What was important, the researchers stress, was the use of suitable teaching approaches to make the ideas accessible to pupils. They suggested that effective teaching required 'pedagogic content knowledge', which incorporated the following three elements:

- understanding of the numeracy knowledge appropriate to what is being taught
- knowledge of how pupils learn numeracy
- understanding of different teaching approaches for presenting information to pupils.

Evidence from the research also indicated that pupil gains in the tests were greatest for teachers who had a good understanding of the conceptual links between the areas of numeracy in the primary curriculum.

How did continuing professional development (CPD) influence teacher effectiveness?: The effect of type and duration of CPD

Data about teachers' experiences of CPD were collected by questionnaire from the full sample of 90 teachers and from interviews with the 33 case study teachers. Of the 88 teachers who responded to the questionnaire, 26% (23 teachers) had undertaken some form of extended CPD.

The researchers compared the mean gains of pupils whose teachers had engaged in mathematics CPD with those whose teachers had received other types of extended CPD. Results suggested that the classes of those teachers who had experience of extended mathematics CPD showed higher gains than those of other teachers including those who had undertaken some form of (non-mathematics) CPD. This finding highlights the importance of pedagogic content knowledge, ie, knowledge of teaching approaches within a specifically mathematics context.

Results also suggested that the length of time spent on CPD had an important bearing on the effectiveness of the CPD in furthering teachers' skills. Analysis of the data showed that only those teachers who had engaged in CPD of at least 15 days duration were highly effective. The class scores of teachers having had three or fewer days of CPD in the previous year were indistinguishable from teachers reporting no days.

Data from the case study teachers indicated that certain aspects of mathematics-focused CPD helped teachers become more effective in putting over concepts to their pupils. Four of the five highly effective connectionist orientated teachers all identified an emphasis on the importance of working with pupils' meanings and understandings as significant elements of their CPD. This had led them to appreciate more fully the role of mental strategies in helping themselves and their pupils build up the mental imagery that would enable them to work through numeracy problems more quickly and efficiently.

One teacher, whose Year 6 class had very much higher gains on the Y5/Y6 test than any other class, reported that her approach to numeracy teaching had changed considerably as the result of the CPD she had undertaken:

"(prior to the CPD)... I would have taught them a set way of doing your long division and the algorithms that I learned at school without bothering about what it means,...but (in a recent topic) they all did it in a way they could understand."

The effect of teacher collaboration in the schools
When the focus schools were ranked according to performance in pupil tests for those pupils at the beginning of Year 2 and then again in line with the results of the tests for their Year 6 pupils at the end of the research, the results showed that two schools in particular seemed to have improved their ranking considerably.

In one of the two schools, key features of practice among the numeracy teachers in the school were:

- strong leadership by the connectionist orientated teachers
- discussion of teaching methods and activities at a more detailed level than seemed to be the case in other schools
- provision of time for key teachers to work collaboratively with other teachers in the classroom.

A flavour of the kind of collaborative activity that teachers in this school found helpful is provided by one of the teachers who led the activities:

"I work closely with each (year group) team so I talk about the work I do and they talk about the work they do. Then we try to pass things around and we have a lot of discussion about the problems the children have, how we can solve them, and I will look for things to support them."

How were teachers selected for the research?: Identifying the 'focus' schools
The focus schools were a group of schools already known to be performing well above expectations in relation to numeracy. The schools came from three local education authorities in London and the southeast of England. The local education authorities were able to provide the research team with extensive school-level data about pupils' performance in numeracy.

Each LEA assisted the researchers in identifying effective focus schools, using a number of data sources which included IQ scores, reading test scores, baseline assessments and national test results. They also included two independent schools. The final sample of six focus schools comprised four state primary schools and two independent preparatory and prepreparatory schools.

The researchers included schools that reflected different socio-economic intakes in different environments - inner city, suburban and rural.

Identifying the teacher sample
The six focus schools and five 'validation' schools (see below) provided 66 and 24 teachers respectively for inclusion in the study. From the six focus schools, 18 teachers - three from each school - were selected by the

researchers for case studies in order to provide data about classroom practices and teachers' beliefs about, and knowledge of, mathematics, pupils and teaching. The three teachers from each school were identified as those most likely to be highly effective. Selection was based on the headteachers' recommendations and, where appropriate, advice from LEA inspectors and advisors. The selection was made so that teachers were evenly distributed across Years 1 to 6.

How did the researchers validate their findings?
The research design raised the important question of validity of findings. Because the teachers in the focus schools were selected on the basis that they were highly effective in teaching numeracy the researchers needed to establish a way to ensure that the findings really did relate to effective teaching and that the same characteristics could be observed in teachers outside the sample.

To ensure as far as possible the validity of their findings about teacher effectiveness, the researchers selected five 'validation schools' in order to provide a variety of effective, average and less effective teachers of numeracy.

Like the focus schools the validation schools were selected based on:

- evidence of performance in the teaching of mathematics, based on national test and other results
- being representative of a range of schools in terms of size, socioeconomic backgrounds of pupils and environments (inner city, suburban and rural).

The researchers also used a range of instruments for collecting different kinds of data to dig under the surface of teachers' practice. For example, one method was the 'personal construct' interview (see next section), which enabled the researchers to supplement other data they had collected.

How was the data collected?
Data about teachers' beliefs and pedagogic content knowledge was collected using a number of methods including:

- a questionnaire administered to all 90 teachers in the focus and the validation schools
- observation of 84 lessons, three for each of the 18 case study teachers in the focus schools and two for each of the 15 teachers in the validation schools
- interviews with the six headteachers of the focus schools
- fifty-four interviews with case study teachers in the focus schools, three for each of the 18 teachers
- an interview with each of the 15 validation teachers
- pupil tests, designed to measure gains in numeracy, at the beginning and end of the six month period of the research.

The questionnaire provided background data, which included the organisation and planning for mathematics teaching and the training and continuing professional development of teachers. It also provided the researchers with data about teachers' perceptions of teaching styles and of their beliefs about teaching, learning and assessing mathematics.

Lesson observation enabled the researchers to examine a number of aspects of teacher practice including organisational and management strategies, teaching styles and pupil responses. Particular foci of attention were the type of instruction provided by the teacher and the use of more sophisticated strategies for calculation as opposed to reliance on counting based methods.

The three interviews consisted of:

- a background interview, which provided evidence to supplement the questionnaire on training and experience, in addition to information about beliefs, knowledge and practices in teaching numeracy
- a 'concept mapping' interview, which explored teachers' mathematical understanding in relation to numeracy teaching

- a 'personal construct' interview, which explored teachers' beliefs and knowledge about the pupils they taught (more details are given below).

The 'personal construct' interviews were important in that they helped the researchers triangulate other data they had collected. They used a series of questions about the mathematical learning of all the pupils in a teacher's class to explore what the teachers regarded as similar and different about the learning of pupils in named groups of three. In this way, the researchers were able to elicit evidence about teachers' beliefs about numeracy in the context of their knowledge about their pupils and about learning processes.

The pupil tests were based on a diagnostic test that had previously been designed and used at King's College. The tests aimed to assess pupils' mental dexterity with numbers and their ability to apply it. The researchers used an aural approach to testing. This had the principal advantages of enabling them to control the timing for certain questions and was helpful for younger children and weaker readers. The bulk of the analysis was based on the performance of pupils in Years 2 to 6 who were tested twice - at the beginning and at the end of the research - rather than on that of Year 1 pupils who were only tested at the end of the research.

What information did the pupil outcome data provide about effective numeracy teachers?
All the classes made gains in the second test. The gains in mean class scores were used to classify the teachers of Years 2 to 6 who participated in the study into the following groups:

- highly effective (23 focus school teachers, 3 validation school teachers)
- effective (15 focus school teachers, 6 validation school teachers)
- moderately effective (17 focus school teachers, 10 validation school teachers).

Only six of the 18 case study teachers, selected on the basis that their headteacher judged them to be highly effective, came into the highly effective category. One reason for this was that nearly half the teachers judged to be highly effective based on pupil test results came from the same school - from which only three teachers had been selected for case study analysis at the outset.

Reasons for other highly effective teachers not being selected for the case study analysis may lie, according to the researchers, in the following:

- some highly effective teachers had been unwilling to participate in the case study work because of other pressures
- the headteacher did not recommend those teachers who were subsequently identified as highly effective
- headteachers did not always recognise the most effective teachers in their schools and knowingly or unknowingly may have proposed teachers for other reasons such as their skills in class control.

What are the implications of the study?
Teachers wanting to improve teaching and learning of numeracy may wish to consider the following implications of the findings of this research review:

- the researchers found that teachers with a 'connectionist' orientation were more likely to have classes that made greater gains than teachers with a 'transmission' or 'discovery' orientation. Would you find it useful to review your own beliefs and approaches to teaching numeracy and consider to what extent they match those of the 'connectionist' teachers the research describes?
- 'connectionist' teachers used formative assessment as a means of identifying what their pupils had learned, where they were struggling and what needed to be covered next. Could you make more use of formative assessment to identify what your pupils need to learn? (You can find out more about formative assessment in two other 'Research for Teachers' summaries: 'Raising standards through classroom assessment' and 'Assessment for learning: putting it into practice'
- effective teachers of numeracy developed their pupils' mathematical thinking. For example, pupils articulated their reasoning verbally or in writing and discussed and evaluated their methods. Could you make more use of this type of approach with your pupils?
- schools in which pupils made good gains in numeracy were found to be those which encouraged open discussion and

opportunities for staff to work together. Would you find it helpful to work collaboratively with your colleagues and share your approaches to teaching? For example would you find it interesting and helpful to explore with your colleagues the connections between the mathematical concepts that you teach?

Leaders may wish to consider the following implications:

- the researchers found that effective teachers of numeracy had access to courses where they learned about successful teaching approaches. Could you make more opportunities available for your staff to participate in professional development activities? (Practitioners may find it helpful to read another of our RfT summaries about the impact of continuing professional development (CPD) upon teaching and learning
- the case studies we have provided came from schools where teachers were working with researchers from higher education institutions on specific projects. Have you considered encouraging teachers in your school to work with a higher education institution to carry out a school-based research project, such as investigating strategies that children use in arithmetic, especially faulty arithmetical procedures that lead to errors, or effective ways of helping pupils overcome difficulties they experience with maths?

Your feedback
Have you found this study to be useful? Have you used any aspect of this research in your own classroom teaching practice? We would like to hear your feedback on this study. To share your views with us email: research@gtce.org.uk
Back to top

Case studies

The following case studies have been selected to illustrate features of teaching and learning in the classroom that are highlighted in the main part of the study. They were not part of the original research project but were separate studies from a number of other schools.

An investigation into pupils' strategies for mental mathematics

This case study illustrates the way in which a thinking skills strategy can help pupils learn mental calculation strategies. The researcher describes how teachers in four first schools implemented a strategy that aimed to promote children's mental and oral skills.

The main components of the intervention were:

- extending the length of the oral/mental start to the lesson by five minutes in three lessons per week
- encouraging the children to articulate, discuss and evaluate the effectiveness of the strategy they used in solving mental calculations
- explicit teaching and modelling of strategies and vocabulary from the national numeracy framework
- conducting individual interviews with pupils so that the teacher could record pupils' explanations of their strategies
- discussing pupils' recorded strategies at professional development meetings
- holding half-termly support group meetings with input from University of Newcastle staff.

By allowing the children more time and encouraging them to articulate their reasoning, the teachers hoped to enable them to clarify their own thinking, benefit from the sharing of strategies with other children and achieve a higher performance.

The formative assessment undertaken during the pupil interviews was highly structured and was based on National Foundation for Educational Research (NFER) methods. This information helped teachers to build up a more complete picture of each child's learning in mathematics.

The findings from the research included the following key features:

- gains in pupils' performance
- improvements in pupils' explanation skills
- increases in pupils' confidence
- use of a greater range of strategies and teaching styles by teachers.

The pupils were tested before and after the intervention using standard tests and there were overall gains which were greater than those expected over a similar period in normal circumstances.

The gains were largest for those pupils who had the lowest standardised scores at the beginning of the project. Staff attributed this result to the more focused teaching they had been able to give the children due to the greater feedback they had obtained about their strengths and weaknesses during the project. However, because the project ran concurrently with the introduction of the National Numeracy Strategy the researcher felt that the gains could not be attributed solely to the intervention, although the teachers believed it had had a major impact.

The researcher found that not only did pupils become more able in articulating their strategies but in many cases, they also showed improved mathematical vocabulary. For example, instead of describing how they calculated, they were able to give the mathematical term for the strategy used. One of the examples, from a Year 4 class, provided by the case study was as follows:

Teacher
560 + 575
Pupil Double 500 is 1,000
Split 75 into 40 + 35
Add 60 and 40, is 100.
Then add 35, is 135.
1,000 and 135 is 1,135
(Strategy - doubles, partitioning)

Perhaps surprisingly, younger pupils showed greater ability in using mathematics vocabulary than older ones. The researcher suggested that this was due to the greater amount of modelling and guidance provided by teachers to the younger age groups.

Teachers also benefited from the intervention. They found the insight they gained from knowledge about pupils' strategies particularly helpful when it came to planning their teaching and they were able to use diagnostic information to identify subsequent steps for learning.

The teachers regarded the CPD, in which they collaborated with their university colleagues, as central to the success of the project. They particularly appreciated the opportunities to:

- discuss numeracy in the context of the national curriculum framework
- consider effective teaching methods
- share their experiences and ideas with others.

Reference: An investigation into pupils' strategies for mental mathematics, Graham Downey, Mickley First School, Mickley, Stocksfield, Northumberland, Teacher Research Grant publication number TPU0657/06-01

Teachers' interpretations of effective whole-class interactive teaching in secondary maths classrooms

This study explores teachers' uses of interactive methods in whole-class teaching to encourage pupil-teacher and pupil-pupil dialogue in secondary mathematics lessons in Welsh schools. Whilst the National Numeracy Strategy does not apply to Wales, some teachers there have taken up ideas for the strategy and have incorporated them into their own practice. This study describes the experiences of one group of Welsh teachers. Whilst the work was conducted in secondary classrooms, it does contain important messages for teachers of mathematics in all key stages.

A major element of the study was its emphasis on reflective discourse in the classroom. The teachers held meetings at which they developed, trialed and evaluated teaching strategies. They used ideas from research including the importance of articulation and scaffolding. The researchers collected data using a number of methods including:

- lesson observations, half of which involved the use of video
- interviews with teachers conducted before and after lesson observations
- in-depth interviews with teachers at the end of the project
- pupil pre- and post-tests, although these have not yet been reported.

During the five months of the project, the teachers encouraged pupils to contribute their ideas and to explain their methods to each other. This example shows how the teacher shared criteria for evaluating methods with the pupils:

Teacher: Give me a number between 2 and one-third and 2 and a half.
Pupil: Miss, 2 and three-eighths.
Teacher (In a non-evaluative tone): How do you know? Can you convince me you are right?

Pupil goes to the board and draws 'fraction cakes' - circles divided roughly into halves, thirds and eighths.

Teacher (To the class): What do you think? Is he right? Are you convinced?

(Some nods from class)

Pupil 2: But... the fraction parts need to be exactly the same size really...
Teacher Yes, they should be, shouldn't they. If you could draw them accurately then maybe that would be OK but with rough skteches on the board I'm not convinced... Can we find a more precise wasy to show it?
Pupil 3 Miss, we could change them to decimals...

(and the pupil is invited to the board to demonstrate this and a similar evaluation follows).

Some of the teachers developed approaches that provided scaffolding and created opportunities for reflection during the lesson. One such strategy involved a 'Start-Stop-Go' activity in which pupils were set a problem and asked to reflect on how to solve it individually. They then worked in groups to generate and evaluate strategies. A whole-class discussion followed, during which the teacher focused pupils' attention on key features and the merits of particular strategies. Finally, the pupils worked in groups to solve the problem.

Another effective strategy was to ask pupils to explain in their own words the method suggested to them by another pupil, which forced them to analyse the explanation before presenting it in their own way. This exercise was carried out in a collaborative way with other pupils making contributions, and the teacher participating to help pupils to focus on key mathematical features.

Pupils were also challenged to identify and correct deliberate mistakes made by the teacher. For example:

"You multiplied instead of squaring so you put (-3)2 = - 6 instead of +9"

The teachers used situations like this to draw attention to common errors and taught their pupils to mark such errors with 'hazard signs' in the margins of their work and to try to predict, at the outset of a task, where such 'danger points' may occur. The teachers felt that this strategy gave pupils a greater sense of ownership and participation:

"My aim was to avoid (pupils) simply being in on the action but to create some of the action for themselves, in order to participate."

Reference: Teachers' interpretations of effective whole-class interactive teaching in secondary mathematics classrooms Sonia Jones and Howard Tanner *Educational Studies*, Vol. 28, No. 3, 2002

Developing skills in mathematical explanation

This case study highlights the importance of children's skills in explaining their reasoning and describes how teachers in one primary school developed children's skills in this area. Whilst the main aim of the study was to develop pupils' skills in writing explanations, the classroom activities incorporated whole class discussion as a major feature.

The researchers found that, after instruction:

- children's mean scores in tests improved
- the children generally wrote more focused explanations, and included more detail
- explanations provided by the children were more complete because the explanations were linked back to the original questions
- in general the children used a more sophisticated mathematical vocabulary
- the children's confidence with symbols improved
- in some cases, children used more verbal explanations unnecessarily.

There was an increase in mean scores for all six topics, including the three for which the children had not received specific instruction in writing explanations - a finding which suggested that the children were able to carry over mathematical ideas and vocabulary between problem areas.

The researchers illustrated the improved ability of children to write explanations using as an example the answers of a Year 6 boy:

Pre-test answer: The answer is 55 - 35 and then you have the answer. Because the drinks are 55p as you can see in the above, so the popcorn is 35p.
Post-test answer (to a similar problem): If an apple and chocolate is 95p and 2 apples and 1 chocolate is £1.20 I worked out what the distance between 95p and £1.20 was and I was left with how much an apple was so I t ook it away from 95p and I was left with the cost of a chocolate.

The children were all drawn from Years 4, 5 and 6 and had to reach a level of prior proficiency before being included in the study. All the children in those years were given a preliminary test and those who achieved scores above 60% - 22 in total took part in the project. The children identified for the study were then placed in a single group for instruction.

Evidence was collected by means of pre- and post-tests. The subject material was taken from the KS2 national curriculum in mathematics and each test consisted of one question from each of the following topics:

- properties of polygons
- sequences of numbers

- graphs and co-ordinates
- solving equations from word problems
- using multiplication facts
- exploring relationships between sets of four numbers.

In the post-test a major difference from the pre-test was that whilst three of the questions were set on topics that had specifically been taught, three questions covered topics not supported by specific instruction.

The intervention took the form of a series of lessons, which not only taught the content but also incorporated a strategy for improving children's written explanation skills. The emphasis in the lessons was on explanation rather than getting the answer right. Typically, the lessons followed the pattern:

- specific instruction in the first three topics
- whole-class discussion focusing on the information required in an explanation
- working through examples as a whole class, including written explanations
- structured worksheets that gave the children the opportunity to tackle problems on their own.

The teachers provided a framework for the children to use for problem- solving and in writing their explanations which consisted of the following elements (as applied to the length of the side of an equilateral triangle):

- classification of the type of object the problem is about (triangles)
- identification of the attributes and properties the pupils know about the objects (triangles have three sides, equilateral triangles have sides of equal length)
- giving values to the properties that are relevant to the problem (in the equilateral triangle ABC, if side AB = 6 cm, then side BC = 6 cm).

The researcher concluded that under the conditions of the intervention children were taught some of the skills necessary to improve their performance on tasks involving written explanations, and that the skills were transferred to other areas of mathematics.

Reference: Developing skills in mathematical explanation Maureen Loomes St. Vincent de Paul RC JMI School Stevenage, Herts. Teacher Research Grant publication number 63/8-99

Sharp Lane Primary School project report

Sharp Lane Primary School was one of a number of schools that participated in the TTA School-based Research Consortium in Leeds between 1998 and 2000. The research in this school focused on the development of mathematical language and on identifying and sharing of effective teaching strategies. This case study report exemplifies classroom strategies for developing children's reasoning skills. It includes an example showing how teachers can successfully build on children's errors to help them reach a better understanding of mathematical methods.

This phase of the research project was conducted in the school years 1998- 99. Staff underwent professional development during which they were introduced to a number of strategies designed to develop mathematical language and mental strategies in children. Teachers also carried out observations on each other's lessons. Following that there was feedback and group discussion. During the intervention phase, the teachers applied the strategies in their classrooms.

Data were collected by a number of methods including:

- baseline assessment of 100 children in Years 1, 3, 4 and 6 using numeracy tests from the National Numeracy Strategy
- teacher questionnaires
- observations during the spring term
- post-testing of children using tests from the National Numeracy Strategy.

Findings from the research included:

- increases in the children's understanding and use of mathematical language
- improvements in pupil performance on both written and mental tests
- increases in teachers' awareness of children's understanding of mathematical concepts and the difficulties they encountered
- increased teachers confidence
- whole class teaching was most successful when all children were involved.

The teacher-researchers planned to develop strategies that they considered would increase the engagement in, and understanding of, mathematical concepts and methods.

One strategy is called 'remodelling up' and aims to enable staff to build on a child's error to present an improved strategy that connects their thinking.

Teacher: The answer is 9. What could the question be?
Child 1: Two add two
Teacher: Would that be right?
Child 1: Err.....
Teacher: Let's write down your 2 + 2. What would that make?
Child 2: 4
Teacher: Good. Now we have 4. What are we trying to make?
Child 3: 9
Teacher: We have a 4 and we need to get to 9. How many more do we need? Child 4: 5
Teacher: Good.

Teacher turns back to child 1.

Teacher: The answer is. What could the question be?
Child 1: 2 + 2 + 5
Teacher: Well done. We got there in the end.

A second strategy developed by the teachers is called 'remodelling down'. Here, from one child's correct but complex response this strategy makes the answer more accessible to other children:

Teacher: We are going to play 'Describe a number'. Today's number is 39.
Child 1: It is 6 squared plus 3.
Teacher: Wow, that's brilliant. Can you come and write it?

Child 1 writes '6 squared + 3 = 39'

Teacher: Does anyone know what 'squared' means?
Child 2: Times by itself.
Teacher: Good.

Teacher writes (6 x 6) + 3

Teacher: Who can describe 39 another way using the word 'squared'?
Child 3: 5 squared add 14.
Teacher: Well done

The teachers concluded that teaching strategies like those outlined above are effective in enabling teachers to differentiate the work in whole class teaching sessions of mental mathematics.

Reference: Sharp Lane Primary School, The Leeds Primary School Research Consortium, part of the TTA School-based Research Consortia Initiative, 1998 - 2000. For summaries of the reports of the four participating consortia see the TDA website.
Back to top

Further reading

Burton, L. (2001), "Research mathematicians as learners - and what mathematics education can learn from them". *British Education Research Journal*, 27 (5), pp.589-99.

Adhami, M., Johnson, D. C., and Shayer, M. (1998). *Thinking maths: The programme for accelerated learning in mathematics*. Oxford: Heinemann Educational Books.

Where can I find out more online?

Association of Teachers of Mathematics
www.atm.org.uk www.atm.org.uk

Cognitive Acceleration in Mathematics Education Project (CAME) project details can be found at the King's College, London website
www.kcl.ac.uk/education

DfES Research informed Practice website presents summaries of a number of areas of educational research including numeracy
www.standards.dfes.gov.uk/research

DfES Standards website provides downloadable resources for teachers to use for the National Numeracy Strategy
www.standards.dfes.gov.uk/numeracy

Raising standards through classroom assessment: a summary of Inside the Black Box
http://www.gtce.org.uk/teachers/rft/assess0501/

The Mathematical Association
www.m-a.org.uk

Related research

Researching effective CPD in mathematics education
www.ncetm.org.uk/enquiry/9251

Effective pedagogy in mathematics Best Evidence Synthesis:
www.educationcounts.govt.nz/__data/assets/pdf_file/0007/7693/BES_Maths07_Complete.pdf

Back to top

Appraisal

This robust study aimed to identify and analyse the knowledge, beliefs and practices of a sample of effective numeracy teachers and to start to explore how they came to be effective. A major feature of the research, which was carried out in primary schools in England, was the use of pupil pre- and post-test data to measure gains in numeracy, which were then correlated with teacher effectiveness.

Other data that enabled the researchers to build up a picture of the characteristics of the effective numeracy teachers were drawn from questionnaire data, observations of teaching and learning in classrooms and interviews with case study teachers. Whilst the researchers recognise the complexity of the factors that contribute to effectiveness of numeracy teaching they nonetheless point to characteristics of teachers which seem to be particularly significant in leading to effectiveness.

Relevance
Numeracy was and remains a topic of major interest to teachers, the public and the Government. At the time of the study, there was a climate of concern about national standards of numeracy, which was reflected in international surveys, Ofsted reports and the Teacher Training Agency. The study was a response to that concern. Whilst the focus was on the primary school, many teachers in other phases will find the description and analysis of teaching styles and their impact on pupils' learning particularly interesting and relevant.

Applicability
Pedagogy lies at the heart of the study. The researchers emphasise that while teachers do need to have sufficient knowledge, the key factors appear to be related more to how teachers engage and maintain the interest of children in their classrooms. The educational contexts in which the research took place will be familiar to many teachers. Informative illustrations are provided to show how teachers approached numeracy teaching and how they engaged pupils in learning numeracy. Teachers in all phases will find the research helpful in enabling them to analyse and build on their own beliefs and strategies.

Writing
The report is written in a lively style that engages the reader's attention. It is free of jargon. The structure of the report is straightforward and clearly separates findings from methodology and discussion. The few technical data contained in the report are described straightforwardly; more complex data are set out in appendices.

Back to top