# Developing Skills in Mathematical Explanation

**Maureen Loomes**
St Vincent de Paul RC JMI School,
Stevenage, Herts

## Aim

To find out ways in which teachers can become more effective at developing children's skills in the area of written mathematical explanation.

## Dimensions of this Case Study

22 children were selected from Years 4, 5 and 6 and tested before and after the period of instruction.

## Summary of Findings for this Case Study

- Children are expected to use mathematical explanation for a variety of purposes, such as rationalising, justifying and clarifying solutions to problems.
- There is little support in published schemes of work for the national curriculum requirement that certain aspects of mathematical explanation should be taught in the curriculum.
- Written mathematical explanation, where children are asked to justify their answers, can be taught as a topic in its own right.
- Children's written mathematical explanation can be supported by teaching them to use a general framework.
- The framework can be used to teach written explanation through specific mathematical topics.
- The children were able to carry across ideas so that their explanations in other mathematical areas, that were not specifically taught, also improved.
- The techniques used have been found to be sufficiently simple to be incorporated into typical current schemes of work.

## Motivation and Background

The National Curriculum requires that certain aspects of mathematical explanation should be included explicitly in the curriculum. For example, at KS2 it is specified that "Pupils should be taught to ...explain their reasoning". The preliminary report of the Numeracy Task Force (DfEE, 1998) also suggests that "numerate pupils should explain their methods and reasoning" and teachers should "collect information about... the clarity of explanations given in oral and written responses." The links between the notions of 'communication', 'understanding' and 'reasoning' are complex and 'explanation' involves aspects of all three.

The recent report on Effective Teachers of Numeracy highlights the importance of the philosophical view that teachers and pupils take of mathematics itself, and the significant impact that this can have for teaching practices in this area (Askew et al, 1997). There are many views one can take of the nature of mathematics, and mathematical knowledge, all of which will have an effect on the teaching of reasoning and communication skills.

> ***"A general framework was used to provide structure for both the teaching and the subsequent explanations."***

The research set out to explore whether there are practical ways in which we can teach some of the required skills for written explanation and to consider how these might be introduced into classroom practice within a typical curriculum.

## The Empirical Study

In this case study 'explanation' was treated as a topic in its own right, to see if it can be explicitly incorporated into schemes of work.

The following specific questions were posed:

- If we limit attention to domains where children can already solve problems, and set closed tasks, can we use a general framework to teach children about explanation so that they improve their performance in terms of demonstrating their understanding?
- If we teach this aspect of written explanation through specific mathematical examples, can the children carry across the ideas so that their performance improves in other types of mathematical problems?

In answering these, we hoped to suggest an answer to the more complex, general, question:

- Is it worth pursuing the question of how to teach mathematical explanation in a more general setting or, as Siegler has suggested, is problem-solving research limited to specific activities?

## The Children

The children for this study were drawn from Years 4, 5 and 6. In order to ensure that they had sufficient mathematical knowledge to solve the problems (since we want them to explain an answer to a problem that they are able to solve), all the children in these year groups were given an Essential Assessment Mathematics Test (McArdle, 1996) and 22 children scoring above 60% were selected for the study. This group had been formed within the school prior to the empirical study, and the children were already used to working together as a class with the researcher as the teacher, even though they were drawn from different year groups.

## The Tasks

The children were given a pre-test before the start of the instruction and a post-test at the end of the period of instruction. Each test consisted of six problems, loosely based on previous KS2 national tests papers in Mathematics. Each problem required the children to explain their answers. In both tests one problem was set on each of the following topics drawn from the KS2 National Curriculum in Mathematics:

- Geometry problems where the properties of polygons were used to find the length of sides from the information given.
- Sequences of numbers where properties of numbers and number patterns were used to find subsequent terms in the sequences.
- Graphs and co-ordinates problems where

properties of co-ordinates on a straight line were used to determine whether a given point was on the line.
- Solving equations to find the price of an object from information given as a word problem.
- Using a given multiplication fact to find the answer to a related multiplication problem.
- Exploring relationships between sets of four numbers presented in a triangular grid as well as patterns in the sequence of 'triangles' to find subsequent terms (i.e. a geometric shape but based on number sequences, relationships).

The problems set were closed tasks that had specific clear cut answers.

The tasks in the post-instruction test were divided into two groups.

- Three tasks supported by specific teaching within the context of the particular problem domain.
- Three tasks not supported by specific instruction in the problem domain, but set after the lessons relating to the tasks in the first group.

Thus for the second three tasks children were required to carry ideas across between problem domains unaided (for example, could they use the ideas for writing explanations of solutions to geometrical problems when explaining the solutions to equations?).

## The Teaching Strategy

A general plan was followed for each lesson:

The aim of focusing on explanation rather than 'getting the right answers' was stressed, and discussed in general terms. The children were given specific instruction in the first three problem areas. For each problem area this started with a whole class discussion focusing on the type of information that might be included in a typical explanation. Examples were worked in a whole-class setting, illustrating written explanations similar to that in the task to follow. Structured worksheets were set that encouraged the children to go through the stages for themselves, and produce explanations to problems similar to those set in the pre- and post-tests.

## The General Framework

A general framework was used to provide structure for both the teaching and the subsequent explanations. This was deliberately chosen so as to be very general and widely applicable:

- classify what types of objects the problem is using (e.g. 2D shapes, triangles, prime numbers, number patterns, fractions...);
- identify attributes and properties that they know about the objects (e.g. triangles have 3 sides, equilateral triangles have sides all the same length, numbers have differences between them...);
- give values to the properties that are relevant to this problem (e.g. in the equilateral triangle ABC, if side AB = 6cm. then side BC = 6cm.)

***"The children generally wrote rather more focused explanations after the instruction and included more detail in their answers."***

## Results

- The children generally wrote rather more focused explanations after the instruction and included more detail in their answers.
- After the instruction they included more examples to support their explanations and included more steps in their explanations, leaving less to be assumed or worked out by the reader.
- Explanations after instruction were more complete because the children remembered to link their explanations back to the original question.
- On occasion, however, more succinct explanations were replaced by rather more verbal but less elegant explanations.
- Some children who had already adopted a symbolic means of expression in the pre-test appeared to be able to use symbols in a more confident way after instruction.
- In general the children used a more sophisticated mathematical vocabulary in the post-test which assisted them in developing their explanations.

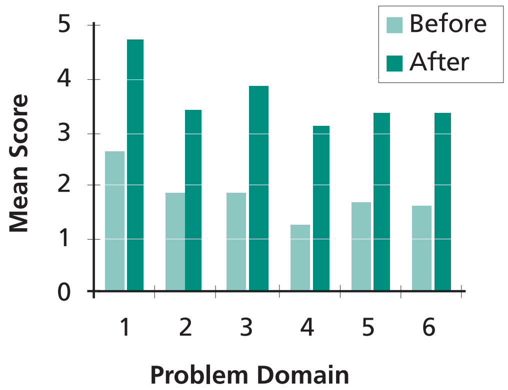
For example, in the pre-test, when answering the question:

A drink and a box of popcorn together cost 90p.
2 drinks and a box of popcorn together cost £1.45.
What does a box of popcorn cost?
a boy in Year 6 gave this explanation:
*"The answer is 55 – 35 and then you have the answer. Because the drinks are 55p as you can see in the above so the popcorn is 35p."*

In the post-test he gave the following explanation for the solution of a similar problem:
*"If an apple and chocolate is 95p and 2 apples and 1 chocolate is £1.20 I worked out what the distance between 95p and £1.20 was and I was left with how much an apple was so I took it away from 95p and I was left with the cost of a chocolate."*

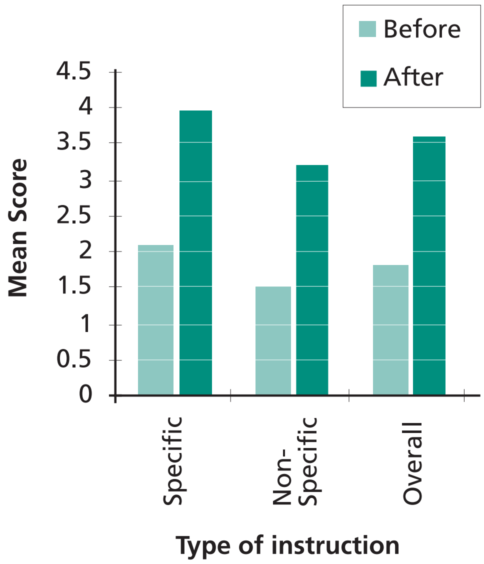
The explanations in the pre and post tests were marked and validated according to criteria laid out in a marking scheme, and up to five marks were given for each written explanation.

Mean scores obtained before and after instruction in each problem domain




This graph shows that there was an increase in the mean scores after instruction. The first three tasks were supported by specific teaching while the second three tasks were set after the lessons relating to the tasks in the first group. Statistical analysis showed that the increase in mean scores was significant for all six tasks.

Mean scores obtained before and after instruction for specific and non-specific problem domains and overall for the six problem domains




The mean scores for specifically taught problems, and for the problems for which the children did not receive specific instruction, together with the mean scores over all six problems are shown in the graph above.

The children's explanations improved even in areas that were not specifically taught.

## Conclusions

The results of our empirical study suggest that, under certain specific conditions, children can be taught some of the skills necessary to improve their performance on tasks requiring written explanations and that these skills can be transferred to other areas of mathematics.

One important question that was carefully monitored was the issue of how well the teaching fitted into the curriculum. The decision to work with just one class of children over a sustained period meant that the material had to be incorporated into a properly managed scheme of work, and not treated as an activity outside of the curriculum. This aspect of the study seemed to work very well. In the traditional mathematical areas upon which the lessons were focused, it provided revision of basic ideas, extended these in natural ways, reinforced the use of correct terminology, provided excellent opportunities for questions, and yet it still offered the opportunity to develop the skills required for explanation.

The children accepted ‘explanation’ as a valid topic for lessons, even though they could already ‘do’ the problems. One Year 6 child subsequently identified ‘explanation’ as an area that he would like to revise before taking the KS2 national tests.

The techniques used in this case study could be incorporated into typical schemes of work in Key Stage 2, using the general framework as a basic structure for developing children’s skills.

## Further Reading

Askew, M., Brown, M., Rhodes, V., Johnson, D., & William, D., *Effective Teachers of Numeracy*, Report of a study carried out for the Teacher Training Agency 1995-6 by the School of Education, King's College London (1997).

DfEE, *Numeracy Matters: The Preliminary Report of the Numeracy Task Force*, Crown Copyright, London (1998).

McArdle, S., *Essential Assessment Mathematics: National Curriculum Practice Tests*, Stanley Thornes, Cheltenham (1996).

Siegler, R. S., *Children's Thinking* (2nd ed), Prentice Hall, Englewood Cliffs (1991).

## Contact

Maureen Loomes, St Vincent de Paul RC JMI, Bedwell Crescent, Stevenage, Herts SG1 1NJ

Publication number 63 / 8–99